Differential Equations and the Parachute Problem

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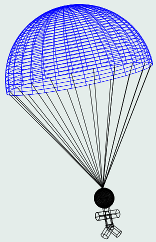
May 10, 2004



Abstract

The parachute problem is a classical first semester differential equations problem often introduced when first learning to solve particle motion problems. The idea of this application is based on a model for the motion of a skydiver when the coefficient of air resistance changes between free-fall and the final steady-state descent with the parachute fully deployed. The authors will present an analysis of this problem, as well as explain why many of the models posed in basic DE textbooks are not physically realistic. We will also propose an improved model that is based on real-life information about skydiving.

1. The Original Model

The basic model of the “parachute problem” can be expressed as an initial value problem for the position, x, or the (vertical) velocity, v, of a skydiver under the forces of gravity and air resistance. The force due to air resistance in this basic model is considered proportional to the velocity.

The mathematical model for motion is provided by Newton’s second law, $F = ma$, with a skydiver of mass m and an acceleration a. F is the sum of a gravitational force F_g and a drag force F_d due to air resistance. Therefore, $F_g + F_d = ma$, where $a = dv/dt$ and $v = dx/dt$. $F_g = -mg$ with $g \approx 9.81$ m/s^2. Assuming the force due to friction, F_d, is proportional to the velocity, we have $F_d = -kv$. The force due to air resistance, k, will be considered in more detail later in the article. For now, it has one value, k_1, when the skydiver is in free-fall and a second value, k_2, when the parachute is fully deployed. If the deployment occurs at time t_0, then we have two cases for drag,

$$k = \begin{cases} k_1, & 0 \le t < t_0 \\ k_2, & t \ge t_0 \end{cases}$$

At this point the problem can be expressed as either a second-order differential equation (ODE) for position or as a first-order system of ODE’s for the velocity and position. The first order is much simpler to solve. During free-fall, the velocity satisfies the initial value problem

$$m\frac{dv}{dt} = -mg - kv, \qquad v(0) = 0, \tag{1}$$

with $k = k_1$. This equation can be solved either as a first-order linear ODE or as a separable ODE. We will present a solution using an integrating factor. Arranging in the form $dv/dt = a(t)v + f(t)$ yields

$$v' = -\frac{k_1}{m}v - g.$$

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Finding the integrating factor:

$$u = e^{-\int a(t)dt}$$
$$u = e^{-\int -k_1/m\, dt}$$
$$u = e^{k_1 t/m}$$

Multiplying by the integrating factor and then integrating we find

$$e^{k_1 t/m}v' + \frac{k_1}{m}e^{k_1 t/m}v = -ge^{k_1 t/m}$$
$$\int (e^{k_1 t/m}v)'dt = \int -ge^{k_1 t/m}dt$$
$$e^{k_1 t/m}v = \frac{-mg}{k_1}e^{k_1 t/m} + C.$$

Solving for C with the initial condition $v(0) = 0$, we find

$$e^{k_1(0)/m}(0) = \frac{-mg}{k_1}e^{k_1(0)/m} + C$$

$$C = \frac{mg}{k_1}.$$

Returning C to the equation and simplifying yields

$$e^{k_1 t/m}v = \frac{-mg}{k_1}e^{k_1 t/m} + \frac{mg}{k_1}$$
$$v(t) = \frac{-mg}{k_1} + \frac{mg}{k_1}e^{-k_1 t/m}$$
$$v(t) = \frac{mg}{k_1}(e^{-k_1 t/m} - 1), \qquad 0 \le t < t_0.$$

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The position can be obtained by integrating the velocity with the initial condition $x(0) = 0$. Therefore,

$$\begin{aligned} y(t) &= \int v(t)dt, \qquad y(0) = y_0 \\ y(t) &= \int -\frac{mg}{k_1} + \frac{mg}{k_1} e^{-k_1 t/m} dt \\ y(t) &= -\frac{mg}{k_1} t - \frac{m^2 g}{k_1^2} e^{-k_1 t/m} + C. \end{aligned}$$

With the initial condition $y(0) = y_0$, the solution becomes

$$\begin{aligned} y(0) &= -\frac{mg}{k_1}(0) - \frac{m^2 g}{k_1^2} e^{-k_1(0)/m} + C \\ y(0) &= -\frac{m^2 g}{k_1^2} + C. \end{aligned}$$

Therefore,

$$C = y_0 + \frac{m^2 g}{k_1^2}.$$

Substituting C into the equation above yields

$$y(t) = y_0 - \frac{mg}{k_1} t - \frac{m^2 g}{k_1^2}(e^{-k_1 t/m} - 1), \qquad 0 \leq t < t_0. \tag{2}$$

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2. Chute Deployment

At times greater than t_0 the chute has been deployed and the drag increases to the value k_2. The solution is essentially the same for the second interval except that the value of drag is now k_2 and the initial condition will be the velocity at time t_0, the solution to the first interval. The solution for the velocity can can be found by solving

$$m\frac{dv}{dt} = -mg - k_2 v, \qquad v(t_0) = \frac{mg}{k_1}(e^{-k_1 t/m} - 1).$$

Arranging in the form $dv/dt = a(t)v + f(t)$ yields

$$v' = -\frac{k_2}{m}v - g.$$

Finding the integrating factor:

$$\begin{aligned} u &= e^{-\int a(t)dt} \\ u &= e^{-\int -k_2/m\, dt} \\ u &= e^{k_2 t/m} \end{aligned}$$

Multiplying by the integrating factor and then integrating we find

$$\begin{aligned} e^{k_2 t/m}v' + \frac{k_2}{m}e^{k_2 t/m}v &= -ge^{k_2 t/m} \\ \int (e^{k_2 t/m}v)'dt &= \int -ge^{k_2 t/m}dt \\ e^{k_2 t/m}v &= \frac{-mg}{k_2}e^{k_2 t/m} + C. \end{aligned}$$

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Therefore, velocity for $t \geq t_0$ is

$$v(t) = -\frac{mg}{k_2} + Ce^{k_2 t/m}.$$

Recall that the velocity for our first interval, $(t \to t_0)$, is

$$v(t) = \frac{mg}{k_1}(e^{k_1 t_0/m} - 1).$$

Since the velocities cannot change instantaneously, the velocities must be the same just prior to and just after t_0. Therefore, $v_{t_0^-} = v_{t_0^+}$. Then

$$\begin{aligned} \frac{mg}{k_1}(e^{-k_1 t_0/m} - 1) &= -\frac{mg}{k_2} + Ce^{-k_2 t_0/m} \\ \frac{mg}{k_2} + \frac{mg}{k_1}(e^{-k_1 t_0/m} - 1) &= Ce^{-k_2 t_0/m} \\ C &= \frac{mg}{k_2}e^{-k_2 t_0/m} + \frac{mg}{k_1}e^{k_2 t_0/m}(e^{-k_1 t_0/m} - 1). \end{aligned}$$

Substituting C into the equation for velocity yields

$$\begin{aligned} v &= -\frac{mg}{k_2} + e^{-k_2 t/m}\left[\frac{mg}{k_2}e^{k_2 t_0/m} + \frac{mg}{k_1}e^{k_2 t_0/m}(e^{-k_1 t_0/m} - 1)\right] \\ v &= -\frac{mg}{k_2} + \frac{mg}{k_2}e^{-k_2 t/m}e^{k_2 t_0/m} + \frac{mg}{k_1}e^{-k_2 t/m}e^{k_2 t_0/m}(e^{-k_1 t_0/m} - 1) \\ v &= \frac{mg}{k_2}(-1 + e^{-k_2 t/m}e^{k_2 t_0/m}) + \frac{mg}{k_1}(e^{-k_2 t/m}e^{k_2 t_0/m})(e^{-k_1 t_0/m} - 1) \\ v &= \frac{mg}{k_2}(e^{k_2(t_0 - t)/m} - 1) + \frac{mg}{k_1}(e^{k_2(t_0 - t)/m})(e^{-k_1 t_0/m} - 1). \end{aligned}$$

Therefore, the equation for velocity is

$$v(t) = \frac{mg}{k_2}(e^{-k_2(t - t_0)/m} - 1) + \frac{mg}{k_1}(e^{-k_2(t - t_0)/m})(e^{-k_1 t_0/m} - 1), \qquad t \geq t_0.$$

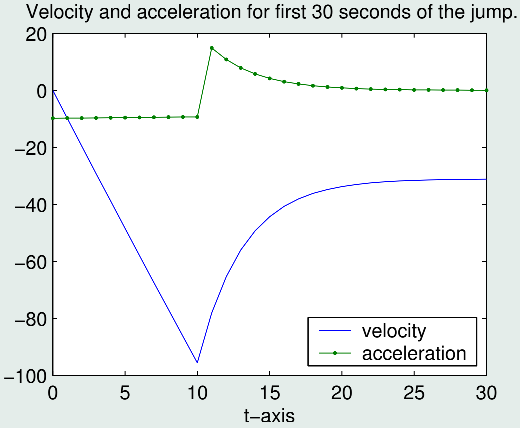

Figure 1: Velocity and Acceleration for Simplified Model

Integrating the equation for velocity ,will of course, yield the equation for position. The velocity at any time t of the two stages of the jump are easy to compute from the equation above as well. The terminal velocity, v_t, is even easier to find by setting $dv/dt = 0$. With zero acceleration, $v_t = -mg/k$.

Figure 1 is a graph of the model of a parachute jump with only two values for drag, k_1 and k_2, where k_2 occurs at the time the parachute is deployed (ripcord pulled) at $t_0 \approx 10$ s. As one can see from looking at figure 1 there is a discontinuity at t_0. Therefore, at t_0 the acceleration is infinite, which realistically cannot be the case. As a result, this simple model is not a very accurate depiction of a real life skydiving jump. Therefore, we will develop a new model that will more accurately describe the motion of a skydiver.

3. Skydiving Physics

To develop a more realistic model of a parachute jump we will need to consider some basic principles of fluid mechanics, principally the relationship between drag and velocity. Essential in this relationship is the connection between viscous and pressure forces. In fluid mechanics this is often accomplished with a non-dimensional variable, called the Reynolds Number. In general $Re = \rho dv/\mu$, where ρ is the fluid density, d is the length normal to the flow, v is the characteristic velocity, and μ is the fluid viscosity. When viscous forces dominate, we have low Reynolds number flows ($Re < 1$), and the drag is linear in velocity. However, when pressure forces dominate ($Re > 10^3$), the drag is quadratic in velocity.[7]

To determine which of these relationships is most appropriate for a skydiver, it is necessary to estimate the Reynolds number. For this model we will assume that the density ρ and viscosity μ are essentially constant at altitudes appropriate for skydiving. Therefore, $\rho \approx 1$ kg/m^3, and $\mu \approx 1.5 \text{ x } 10^{-5}$ kg/m/s.[4] The characteristic velocity will be defined by the terminal velocity. With the chute fully deployed, it is frequently said to be equivalent to $v \approx 5.3$ m/s.[5] During free-fall $v_t \approx 45$m/s ≈ 100 mile/hr. The fully deployed parachute presents a cross section of $A \approx 44$ m^2 giving $d \approx 7.5$ m. A skydiver in a horizontal position presents a cross-section to the flow of $A \approx 0.5$ m^2 giving $d \approx 0.8$ m. Thus, $Re > 10^6$ before and after chute deployment and the force due to the drag can be estimated by

$$F_d = \frac{1}{2}(C_d A \rho v^2).[1]$$

C_d, the coefficient of drag, is determined by the shape of the body and is usually found experimentally. Table 1 provides the necessary values for our model. Drag forces are produced by the skydiver's body, the suspension lines, and the canopy.

Parachute deployment is accomplished by various methods. We will, however, model the "lines-first" release where the parachute remains in a deployment bag until the risers and suspension lines are fully extended. This can be modelled in four distinct stages. First, the ripcord is pulled at $t = t_0$. In between t_0 and t_1 is the period it takes for the extension

Shape	Reynolds Number	C_d
Hemispherical Shell	$Re > 10^3$	1.33
Flat Strip	$Re > 10^3$	1.95
Cylinder	$Re > 5 \text{ x } 10^5$	≈ 0.35

Table 1: Drag Coefficients of common Shapes.

lines of the parachute to extend to their full length. The snatch force, the force created by the opening parachute, occurs at $t = t_1$. This force pulls the skydiver from the horizontal spread eagle position to the up right vertical position. At time $t = t_2$, the parachute is fully inflated. Between the times t_2 and t_3, during the deceleration of the skydiver, the parachute experiences over inflation due to the force of the surrounding air. Finally, at t_3, the parachute reaches its steady-state area, a_1, for the remainder of the jump.

Throughout the multiple stages of the jump it is important to realize that both the skydiver's body and equipment generate separate drag forces. The total drag force is

$$F_d = F_d^b + F_d^e = \frac{1}{2}\rho(C_d^b A^b + C_d^e A^e)v^2$$

where the superscripts b and e are used to denote the different drag coefficients and cross-sectional areas of the skydiver's body and equipment.

In order to finalize the model we need estimates of the parameters discussed in the model. We will be representing an average skydiver with a height of $5'10''$ ($h = 1.778$ m) weighing 190 lbs. The weight of the parachute and suspension lines is 13.85 lbs and the harness is 10 lbs, creating a total mass of 97.2 kg. In the horizontal position, experienced during free-fall, the body can be modelled as a flat rectangular strip with area $b_0 = 0.5$ m^2. In the vertical position, after the parachute opens, the body can be modelled as cylinder with area $b_1 = 0.1$ m^2. The remaining cross-sectional areas and information for the equipment are provided in Table 2, and are taken from real life information. It is now important to discuss the length of each time interval for the multiple stages of the jump.

a_1	b_0	b_1	h	l	m	t_0	t_1	t_2	t_3
43.8 m^2	0.5 m^2	0.1 m^2	1.78 m	8.96 m	97.2 kg	10 s	10.5 s	11.5 s	13.2 s

Table 2: Parameter Estimates for a Typical Skydiver and Equipment.

The times are based on averages taken from real life jumps performed by the United States Air Force Academy at an altitude of 4000 ft. The free-fall lasts for a total of 10 seconds making $t_0 = 10$ s. Independent of the value of t_0 the Snatch force occurs around $t_1 - t_0$ which is 0.5 s after the ripcord is pulled at t_0. The force of the opening parachute occurs at $t_2 - t_1 = 1.0$ s. The complete time for this type of deployment to occur after the ripcord is pulled is 3.2 s. Therefore, $t_3 - t_2 = 1.7$ s. The entire jump from free-fall to steady-state of the parachute lasts for 13.2 seconds.

The following is a summary of the definitions of the cross-sectional areas and drag coefficients for the body and equipment at any specific time during the jump. As well as a piece-wise function for k, a compilation of the summary of drag forces, we have also included figure 2 and 3 to help correlate the information found in the cases with the different stages of the jump.

The area of the body of the skydiver is defined by the following cases:

$$A^b(t) = \begin{cases} b_0, & t \le t_0 \\ b_0, & t_0 < t \le t_1 \\ b_1, & t_1 < t \le t_2 \\ b_1, & t_2 < t \le t_3 \\ b_1, & t > t_3 \end{cases}$$

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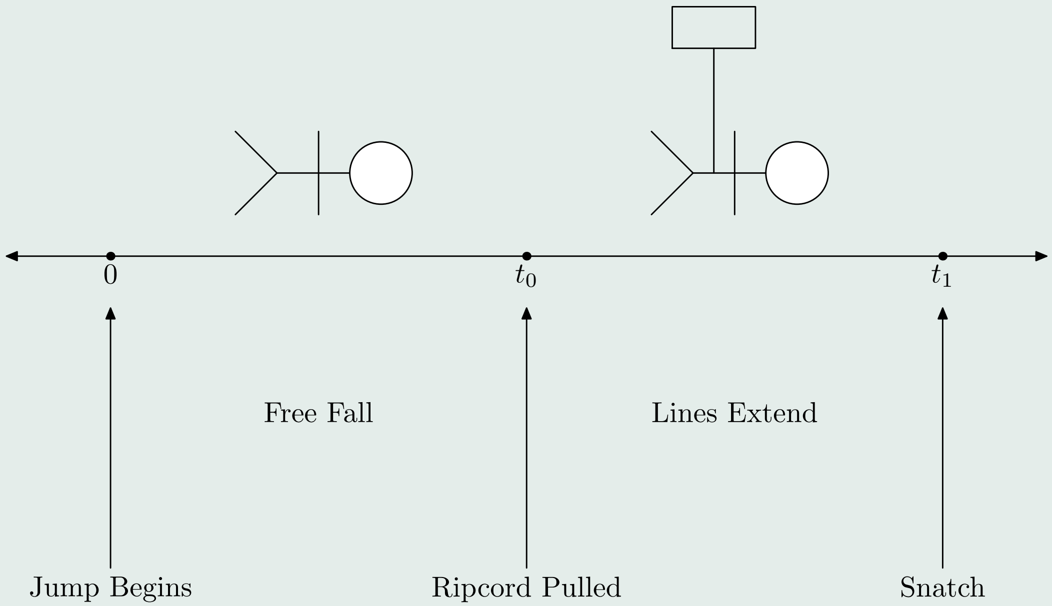

Figure 2: Time-line for Initial Stages of Jump.

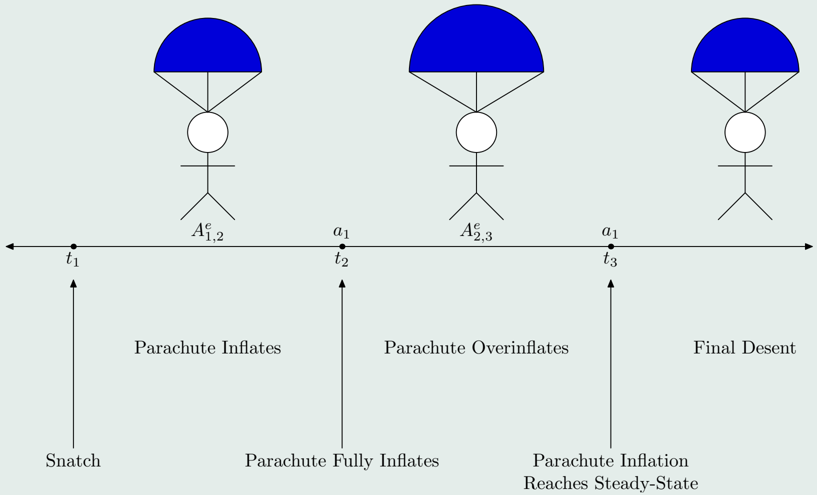

Figure 3: Time-line for Final Stages of Jump.

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The coefficient of drag for the body is defined by the following cases.

$$C_d^b(t) = \begin{cases} 1.95, & t \leq t_0 \\ 1.95, & t_0 < t \leq t_1 \\ 0.35h, & t_1 < t \leq t_2 \\ 0.35h, & t_2 < t \leq t_3 \\ 0.35h, & t > t_3 \end{cases}$$

The area of the equipment for each case is as follows. Note that between the periods t_1 and t_3, the area of the parachute is changing while it inflates and is therefore dependent on time.

$$A^e(t) = \begin{cases} 0.0, & t \leq t_0 \\ b_1, & t_0 < t \leq t_1 \\ A^e_{1,2}(t), & t_1 < t \leq t_2 \\ A^e_{2,3}(t), & t_2 < t \leq t_3 \\ a_1, & t > t_3 \end{cases}$$

The coefficient of drag for the equipment is as follows. Note that from t_0 to t_1, C_d is a function of time, as the lines increase in length as the parachute is deployed.

$$C_d^e(t) = \begin{cases} 0.0, & t \leq t_0 \\ 0.35l\frac{t-t_0}{t_1-t_0}, & t_0 < t \leq t_1 \\ 1.33, & t_1 < t \leq t_2 \\ 1.33, & t_2 < t \leq t_3 \\ 1.33, & t > t_3 \end{cases}$$

Now that we have established all the necessary parameters we can apply them to a new model.

4. Improved Model

The improved model for the velocity of the skydiver is the nonlinear initial value problem:

$$m\frac{dv}{dt} = -mg + kv^2, \qquad v(0) = 0$$

where $k = (1/2)\rho(C_d^b A^b + C_d^e A^e)$

$$k = \frac{1}{2}\rho \begin{cases} 1.95b_0, & t \leq t_0 \\ 1.95b_0 + 0.35b_1 l \frac{t-t_0}{t_1-t_0}, & t_0 < t \leq t_1 \\ 0.35b_1 h + 1.33A^e_{1,2}(t), & t_1 < t \leq t_2 \\ 0.35b_1 h + 1.33A^e_{1,2}(t), & t_2 < t \leq t_3 \\ 0.35b_1 h + 1.33a_1, & t > t_3 \end{cases}$$

To solve the initial value problem analytically for velocity on $(0, \infty)$ we would first have to solve the IPV on $(0, t_0)$ and then use this solution as the initial condition for the next time interval (t_0, t_1). The process for finding differentiable solutions to the remaining intervals, (t_1, t_2), (t_2, t_3), and (t_3, ∞) are done in the same manner. As one can easily see, this process is lengthy and fairly difficult. Therefore, we will use a numerical solver and a piece-wise function found by assembling all of the solutions from the cases in the previous section. Although the solutions for each case are continuous, they may fail to be at any one of the end points. Therefore, it is necessary to prove continuity and differentiability not only over each subinterval of k, but also at their respectable end points t_0, t_1, t_2, and t_3. To do this we will take the limits of each subinterval as they approached their end points from above and below and set the limits equal to each other. This will ensure that the function is smooth over its endpoints as well as differentiable. Therefore, the conditions necessary for our funtion to be continuous are as follows:
At t_0,

$$1.95b_0 = 1.95b_0 + 0.35b_1 l \left(\frac{t - t_0}{t_1 - t_0}\right)$$

Substituting t_0 for t, we have

$$1.95b_0 = 1.95b_0 + 0.35b_1 l \left(\frac{t_0 - t_0}{t_1 - t_0} \right)$$
$$1.95b_0 = 1.95b_0.$$

At t_1, $t = t_1$, then

$$1.95b_0 + 0.35b_1 l = 0.35b_1 h + 1.33A^e_{1,2}(t_1)$$
$$1.95b_0 + 0.35b_1 l - 0.35b_1 h = 1.33A^e_{1,2}(t_1)$$
$$1.95b_0 + 0.35b_1(l - h) = 1.33A^e_{1,2}(t_1)$$
$$A^e_{1,2}(t_1) = \frac{1.95b_0 + 0.35b_1(l - h)}{1.33}.$$

At t_2, $t = t_2$, then

$$0.35b_1 h + 1.33A^e_{1,2}(t_2) = 0.35b_1 h + 1.33A^e_{2,3}(t_2$$
$$A^e_{1,2}(t_2) = A^e_{2,3}(t_2).$$

Note: The definition of t_2 is the time when the opening shock is felt. This implies that the cross-sectional area is a_1, the final steady-state condition.
At t_3, $t = t_3$

$$0.35b_1 h + 1.33A^e_{2,3}(t_3) = 0.35b_1 h + 1.33a_1$$
$$A^e_{2,3}(t_3) = a_1.$$

From the above calculations it is fairly easy to tell that the continuity of k is ensured by the continuity of $A^e_{1,2}$ and $A^e_{2,3}$ on their respective subintervals. And further, that the linear function used to model the extension of the suspension lines ensures continuity at t_0. The continuity at t_1 requires that

$$1.95b_0 + 0.35b_1 l = 0.35b_1 h + 1.33A^e_{1,2}(t_1).$$

The continuity at t_2 calls for

$$A^e_{1,2}(t_2) = A^e_{2,3}(t_2).$$

Finally, the continuity at t_3 requires that

$$A^e_{2,3}(t_3) = a_1.$$

Experimental data for the canopy area during deployment is presented in [6],(p. 246 Figure 6.10B). The area appears to be essentially exponential. Therefore, we will let

$$A^e_{1,2}(t) = \alpha_0 e^{\beta_0 (t-t_1)/(t_2-t_1)}.$$

The conditions established above are satisfied when

$$\alpha_0 = \frac{1.95 b_0 + 0.35 b_1 (l-h)}{1.33}$$

and

$$\beta_0 = \ln \frac{a_1}{\alpha_0}.$$

Specific information about the over-inflation is more difficult to obtain. One possible function, suggested by Meade in [1], satisfying our conditions is

$$a_1 = A^e_{2,3}(t) = \alpha_1 \left(1 + \beta_1 \sin\left(\pi \frac{t-t_2}{t_3-t_2}\right)\right).$$

where the parameter β_1 represents the relative increase in area above the nominal area of the parachute. Experimental data suggests that the parachute over-inflates to approximately 115 percent of its original area, making $\beta = 0.15$ a reasonable choice.[1]

The model and values for all of its parameters are now completely determined. A numerical solution of the problem can be created and graphed using a software package such as MATLAB (See Appendix).

5. Verification of the Model

Prior to looking at a numerical approximation of the solution, recall that the terminal velocity can be determined from

$$v_t = -\sqrt{\frac{mg}{k}} = -\sqrt{\frac{2mg}{\rho(C_d^b A^b + C_d^e A^e)}}.$$

With k as defined in our piece-wise cases and the numerical parameters given previously, the free-fall terminal velocity is $v_t \approx -44.2$ m/s≈ -98.9 miles/hr while the impact velocity should be approximately $v_t \approx -5.72$ m/s. The free-fall terminal velocity is exceptionally close to the 100 mile/hr estimate given previously. Figure 4 provides additional verification of terminal velocity and smoothness results. The spike in the acceleration contains both the snatch force and opening force. Also, notice that once the motion approaches terminal velocity, the position is essentially linear.

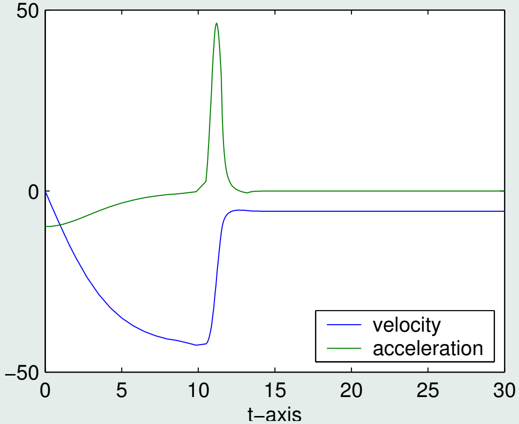

Figure 4: Velocity and Acceleration for the First 30 Seconds.

6. Conclusion

While the new model is an improvement over the one found traditionally in textbooks and classrooms, it does not include all of the physics. Parameters such as changing air density, parachute porosity, and suspension line elasticity all affect the model to some extent. However, significant parameters such as the changing cross-sectional areas and the relationship between resistance and velocity have greatly improved the basic model.

References

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7. Appendix

```
function plot

%Closes all open figure windows.
close all

% The steady-state cross sectional area of chute.
a_1=43.8;

% The cross sectional area of the skydiver in
% horizontal and vertical positions.
b_0=0.5; b_1=0.1;

% The height of the skydiver.
h=1.778;

%The length of the suspension lines.
l=8.96;

%Total mass of skydiver and equipment.
m=97.2;

%Time values for each stage of the jump.
t_0=10; t_1=10.5; t_2=11.5; t_3=13.2;

%Total time span.
tspan=[0,30];

%Initial condition v(0)=0.
```

```
init=0;

%The gravity constant.
g=9.81;

%Maximum cross-sectional area for over-inflation of parachute.
beta_1=0.15;

%Limit as k_3 approaches t_1.
alpha_0=1.95*b_0+0.35*b_1*(l-h)/1.33;

%Allows the limit as k_3 approaches t_2 to be a_1.
beta_0=log(a_1/alpha_0);

%Using a solver to calculate the solution.
[t,v]=ode45(@f,tspan,init,[],a_1,b_0,b_1,h,l,m,...
    t_0,t_1,t_2,t_3,g,beta_0,beta_1,alpha_0);

%Equations for the cross-sectional areas of the parachute during the
%inflation and over-inflation period.
A_1=alpha_0*exp(beta_0*(t-t_1)/(t_2-t_1));
A_2=a_1*(1+beta_1*sin(pi*(t-t_2)/(t_3-t_2)));

%Calculating coefficients of drag.
k_1=0.5*(1.95*b_0);
k_2=0.5*(1.95*b_0+0.35*b_1*l*((t-t_0)/(t_1-t_0)));
k_3=0.5*(0.35*b_1*h+1.33*A_1); k_4=0.5*(0.35*b_1*h+1.33*A_2);
k_5=0.5*(0.35*b_1*h+1.33*a_1);

%Calculation of acceleration.
```

```
a=(-g+k_1.*v.^2/m).*(t<=t_0)+(-g+k_2.*v.^2/m).*(t_0<t &
t<=t_1)+(-g+k_3.*v.^2/m).*(t_1<t & t<=t_2)+...
    (-g+k_4.*v.^2/m).*(t_2<t & t<=t_3)+(-g+k_5.*v.^2/m).*(t>=t_3);

%Plotting the velocity and acceleration.
plot(t,v,t,a)

%Labelling and insertion of a legend for graph of velocity and acceleration.
legend('velocity','accleration',4) xlabel('t-axis')
title('Velocity and Acceleration for 30 second jump')

%Creation of function file.
function
vprime=f(t,v,a_1,b_0,b_1,h,l,m,t_0,t_1,t_2,t_3,g,beta_0,beta_1,alpha_0)

%Equations for the cross-sectional area during inflation and over-inflation
A_1=alpha_0*exp(beta_0*(t-t_1)/(t_2-t_1));
A_2=a_1*(1+beta_1*sin(pi*(t-t_2)/(t_3-t_2)));

%Coefficients of drag.
k_1=0.5*(1.95*b_0);
k_2=0.5*(1.95*b_0+0.35*b_1*l*((t-t_0)/(t_1-t_0)));
k_3=0.5*(0.35*b_1*h+1.33*A_1); k_4=0.5*(0.35*b_1*h+1.33*A_2);
k_5=0.5*(0.35*b_1*h+1.33*a_1);


if (t<=t_0) vprime=-g+k_1*v.^2/m; elseif (t_0<t & t<=t_1)
vprime=-g+k_2*v.^2/m; elseif (t_1<t & t<=t_2)
vprime=-g+k_3*v.^2/m; elseif (t_2<t & t<=t_3)
```

```
vprime=-g+k_4*v.^2/m; elseif (t>t_3) vprime=-g+k_5*v.^2/m;

end
```