**Humboldt State University**
**Mathematics Department**
**Math 115—Precalculus**

# Final Exam
# Pretest

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**Multiple Choice Questions**

**Directions**: *In each of the following exercises, select the "best" answer and darken the corresponding oval on your scantron sheet.*

**1.** One $x$-intercept of the parabola $y = x^2 - 2x - 2$ is

(a) $1 + \sqrt{2}$ (b) $2 - \sqrt{2}$ (c) $3 - \sqrt{3}$

(d) $1 - \sqrt{3}$ (e) $2 + 2\sqrt{2}$

**2.** What is the maximum value attained by the function defined by $f(x) = 4 - 2x - x^2$?

(a) 4 (b) 3 (c) 5

(d) 2 (e) 1

**3.** What is the equation of the axis of symmetry for the parabola defined by $y = 2x^2 - x - 1$?

(a) $y = 1/4$ (b) $y = -1/2$ (c) $x = 1/2$

(d) $x = -1/2$ (e) $x = 1/4$

**4.** If $-2$ is a zero of the polynomial $p(x)$, then which of the following is a factor of $p(x)$?

(a) $x - 2$ (b) $x + 2$ (c) $2x + 1$

(d) $2x - 1$ (e) None of these

**5.** What is the remainder when $x^3 + 2x - 1$ is divided by $x - 2$?

(a) 7 (b) $-2$ (c) 5

(d) $-4$ (e) 11

**6.** Simplify $(2 + 3i)(3 - 4i)$.

(a) $2 + 3i$ (b) $18 + i$ (c) 18

(d) $6 - 12i$ (e) $4 + 3i$

**7.** If $z = 3 + 4i$, evaluate $z\overline{z}$.

(a) $9 - 16i$ (b) $16 - 9i$ (c) 7

(d) 25 (e) $3 - 4i$

**8.** Place

$$\frac{1+i}{1-i}$$

in the form $a + bi$.

(a) $1 + 2i$ (b) $2 - i$ (c) $1 - i$

(d) $1 - 2i$ (e) $i$

**9.** One zero of $y = x^2 - 2x + 2$ is

(a) $i$ (b) $-i$ (c) $1 + 2i$

(d) $1 - i$ (e) $2 - i$

**10.** The rational function

$$f(x) = \frac{x^2 - 2x - 3}{x + 5}$$

has a slant asymptote. What is its equation?

(a) $y = x - 7$ (b) $y = x + 7$ (c) $y = x + 3$

(d) $y = x - 5$ (e) $y = x + 1$

**Essay Questions**

**Instructions**: *This file contains questions form the last module, polynomials and rational functions. In addition, you should study all questions in pretests and examinations for the first three modules; i.e., exams 1 through 3.*

EXERCISE 1. Consider the parabola defined by $y = 5 - 4x - x^2$.

(a) Place the equation in vertex form.

(b) Find the $y$-intercept of the parabola.

(c) Find the $x$-intercepts of the parabola.

(d) Sketch the parabola on a sheet of graph paper. Label the vertex and intercepts with their coordinates. Label the axis of symmetry with its equation.

EXERCISE 2. Consider the function

$$f(x) = x^3 - 6x^2 - x + 30.$$

Without the aid of technology, find the zeros and sketch the polynomial on a sheet of graph paper.

EXERCISE 3. Consider the rational function

$$f(x) = \frac{x+1}{3-x}.$$

(a) Identify the zeros of $f$.

(b) Identify the vertical asymptotes.

(c) Identify the horizontal asymptote.

(d) Without the aid of technology, sketch the graph of $f$ on a sheet of graph paper. Label the zeros with their coordinates, and the asymptotes with their equations.

# Solutions to Quizzes

**Solution to Question 1:** Use the quadratic formula to find the $x$-intercepts of $y = x^2 - 2x - 2$.

$$\begin{aligned} x &= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-2)}}{2(1)} \\ x &= \frac{2 \pm \sqrt{12}}{2} \\ x &= \frac{2 \pm 2\sqrt{3}}{2} \\ x &= 1 \pm \sqrt{3} \end{aligned}$$

□

**Solution to Question 2:** The graph of $f(x) = 4 - 2x - x^2$ is a parabola that opens downward. It achieves its maximum value at its vertex. The $x$-value of the vertex is

$$\begin{aligned} x_v &= -\frac{b}{2a} \\ &= -\frac{-2}{2(-1)} \\ &= -1. \end{aligned}$$

Substitute $x_v = -1$ to find the maximum function value.

$$\begin{aligned} f(-1) &= 4 - 2(-1) - (-1)^2 \\ &= 4 + 2 - 1 \\ &= 5 \end{aligned}$$

□

**Solution to Question 3:** The axis of symmetry is a vertical line through the vertex of the parabola. The $x$-value of the vertex of $y = 2x^2 - x - 1$ is

$$\begin{aligned} x_v &= -\frac{b}{2a} \\ x_v &= -\frac{-1}{2(2)} \\ x_v &= \frac{1}{4} \end{aligned}$$

Therefore, the equation of the axis of symmetry is $x = 1/4$. □

**Solution to Question 4:** If $-2$ is a zero, then we know that $p(x)$ factors

$$p(x) = (x + 2)q(x),$$

where $q$ is another polynomial. Thus, $x + 2$ is a factor of $p(x)$. □

**Solution to Question 5:** Note that $x^3 + 2x - 1$ is equal to $x^3 + 0x^2 + 2x - 1$, requiring a zero placeholder in the synthetic division.

$$\begin{array}{c|cccc} \boxed{2} & 1 & 0 & 2 & -1 \\ & & 2 & 4 & 12 \\ \hline & 1 & 2 & 6 & 11 \end{array}$$

Thus, the remainder is 11. □

**Solution to Question 6:** Multiply in the usual manner.

$$(2 + 3i)(3 - 4i) = 6 - 8i + 9i - 12i^2$$

Recall that $i^2 = -1$.

$$\begin{aligned}(2+3i)(3-4i) &= 6 - 8i + 9i - 12(-1)\\ &= 6 - 8i + 9i + 12\\ &= 18 + i\end{aligned}$$

□

**Solution to Question 7:** Recall that $\overline{z}$ calls for the conjugate of $z$. Thus,

$$\begin{aligned}z\overline{z} &= (3+4i)\overline{(3+4i)}\\ &= (3+4i)(3-4i)\end{aligned}$$

Use the difference of squares pattern to multiply.

$$\begin{aligned}&= 9 - 16i^2\\ &= 9 - 16(-1)\\ &= 25\end{aligned}$$

□

**Solution to Question 8:** Multiply numerator and denominator by the conjugate of the denominator.

$$\begin{aligned}\frac{1+i}{1-i} &= \frac{(1+i)(1+i)}{(1-i)(1+i)}\\ &= \frac{1+2i+i^2}{1-i^2}\\ &= \frac{1+2i+(-1)}{1-(-1)}\\ &= \frac{2i}{2}\\ &= i\end{aligned}$$

□

**Solution to Question 9:** Use the quadratic formula to find a zero of $y = x^2 - 2x + 2$.

$$\begin{aligned}x &= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)}\\ x &= \frac{2 \pm \sqrt{-4}}{2}\\ x &= \frac{2 \pm 2i}{2}\\ x &= 1 \pm i\end{aligned}$$

□

**Solution to Question 10:** Divide $x^2 - 2x - 3$ by $x + 5$.

$$\begin{array}{c|ccc} \boxed{-5} & 1 & -2 & -2\\ & & -5 & 35\\ \hline & 1 & -7 & 32\end{array}$$

Thus,

$$f(x) = x - 7 + \frac{32}{x+5}.$$

In the limit, as $x \to \pm\infty$, $32/(x+5)$ approaches zero. That is, for $x$-values with large magnitudes,

$$x - 7 + \frac{32}{x+5} \approx x - 7.$$

Thus, $y = x - 7$ is a slant asymptote. $\square$

# Solutions to Exercises

**Exercise 1(a)** Add $-5$ to both sides of the equation, then factor a $-1$ from each $x$-term.

$$y - 5 = -(x^2 + 4x)$$

Thake 1/2 of 4 and square the result. Balance by adding $-4$ to the other side of the equation.

$$\begin{aligned} y - 5 - 4 &= -(x^2 + 4x + 4) \\ y = 9 &= -(x + 2)^2 \end{aligned}$$

□

**Exercise 1(b)** Let $x = 0$. Then,

$$\begin{aligned} y &= 5 - 4(0) - (0)^2 \\ y &= 5 \end{aligned}$$

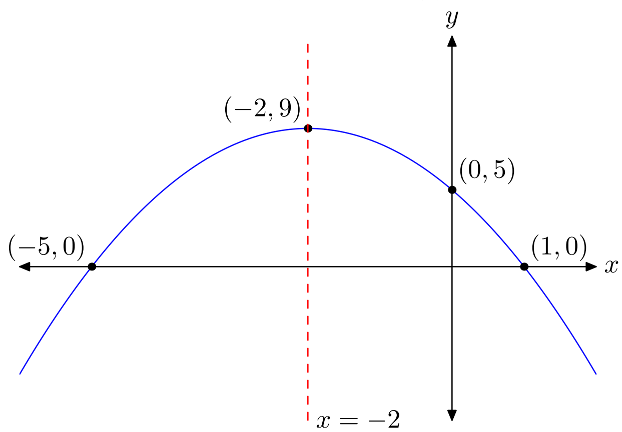
□

**Exercise 1(c)** Let $y = 0$. Then,

$$\begin{aligned} 0 &= 5 - 4x - x^2 \\ 0 &= x^2 + 4x - 5 \\ 0 &= (x + 5)(x - 1) \\ x &= -5, 1 \end{aligned}$$

□

**Exercise 1(d)** Using the information from parts (a) through (c),




The axis of symmetry is a vertical line through the vertex, having equation $x = -2$. □

**Exercise 2.** According to the rational root theorem, if $p/q$ is a rational root, then $p$ divides 30 and $q$ divides 1. Thus,

$$\begin{aligned} p &= \pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 10, \pm 15, \pm 30 \\ q &= \pm 1 \end{aligned}$$

Possible rational roots are:

$$\frac{p}{q} = \pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 10, \pm 15, \pm 30.$$

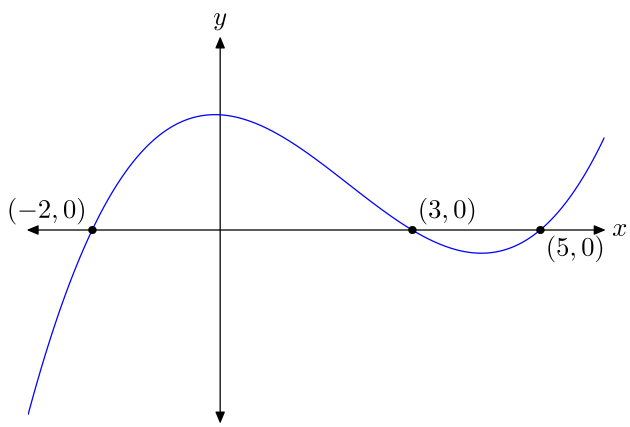
After some experimentation, we found that $-2$ is a zero.

$$\begin{array}{r|rrrr} \boxed{-2} & 1 & -6 & -1 & 30 \\ & & -2 & 16 & -30 \\ \hline & 1 & -8 & 15 & 0 \end{array}$$

Thus,

$$\begin{aligned} f(x) &= (x+2)(x^2-8x+15) \\ &= (x+2)(x-3)(x-5), \end{aligned}$$

and $-2$, 3, and 5 are zeros. The leading term is $x^3$, so the polynomial must mimic the end-behavior of $y = x^3$. Thus,




Exercise 2

**Exercise 3(a)** Because

$$f(x) = \frac{x+1}{3-x}$$

is reduced to lowest terms, whatever makes the numerator zero will be a zero of $f$. Thus, $x = -1$ is a zero of $f$. □

**Exercise 3(b)** Again, because $f$ is reduced to lowest terms, whatever makes the denominator zero will place a vertical asymptote in the graph. Thus, there is a vertical asymptote having equation $x = -3$. □

**Exercise 3(c)** First, divide numerator and denominator by $x$.

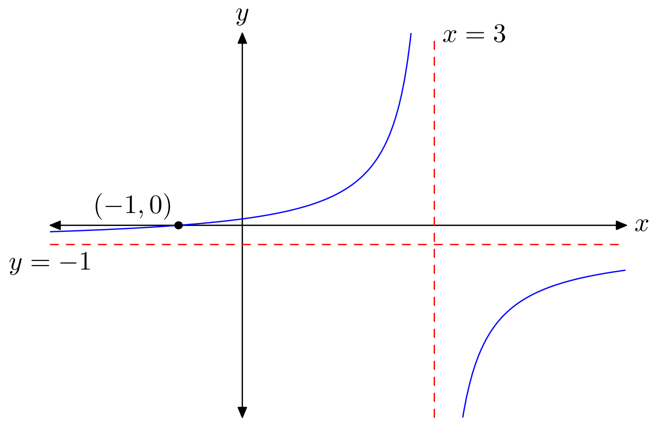
$$\begin{aligned} f(x) &= \frac{x+1}{3-x} \\ &= \frac{\frac{x+1}{x}}{\frac{3-x}{x}} \\ &= \frac{1+\frac{1}{x}}{\frac{3}{x}-1} \end{aligned}$$

Now, investigate the limit as $x$ approaches $\pm\infty$.

$$\begin{aligned} \lim_{x\to\pm\infty} f(x) &= \lim_{x\to\pm\infty} \frac{1+\frac{1}{c}}{\frac{3}{x}-1} \\ &= \frac{1+0}{0-1} \\ &= -1 \end{aligned}$$

Thus, $y = -1$ is a horizontal asymptote. □

**Exercise 3(d)**


y
x = 3
(−1, 0)
x
y = −1

□