College of the Redwoods
Mathematics Department
Math 45—Linear Algebra

Quiz #2—Linear Algebra

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EXERCISE 1. Use elimination and back substitution to solve the following system of equations.

$$x_1 + 2x_2 - 3x_3 = -12$$
$$2x_1 + 6x_2 + x_3 = -7$$
$$-x_1 + 2x_2 - x_3 = -8$$

EXERCISE 2. What matrices E_{21}, E_{31}, and E_{32} put the coefficient matrix in Exercise 1 into triangular form U? Multiply those E's to get one matrix M that does elimination: $MA = U$.

Solutions to Exercises

Exercise 1. Using the coefficients and the answers of the rows to create an augmented matrix,

$$A' = \begin{pmatrix} 1 & 2 & -3 & -12 \\ 2 & 6 & 1 & -7 \\ -1 & 2 & -1 & -8 \end{pmatrix}$$

Use the pivot in row 1 to cancel out the pivots in rows 2 and 3. First, subtract 2 times row 1 from row 2, then add 1 times row 1 to row 3: $R_2 - 2R_1$, $R_3 + R_1$.

$$\begin{pmatrix} 1 & 2 & -3 & -12 \\ 0 & 2 & 7 & 17 \\ 0 & 4 & -4 & -20 \end{pmatrix}$$

Now, use the pivot in row 2 to cancel out the pivot in row 3: $R_3 - 2R_2$.

$$\begin{pmatrix} 1 & 2 & -3 & -12 \\ 0 & 2 & 7 & 17 \\ 0 & 0 & -18 & -54 \end{pmatrix}$$

This augmented matrix represents the following system of equations.

$$\begin{aligned} x_1 + 2x_2 - 3x_3 &= -12 \\ 2x_2 + 7x_3 &= 17 \\ -18x_3 &= -54 \end{aligned}$$

Use back substitution to solve for x_1, x_2, x_3. First, solve the third equation above for x_3.

$$\begin{aligned} -18x_3 &= -54 \\ x_3 &= 3 \end{aligned}$$

Next, plug x_3 into equation 2 and solve for x_2.

$$\begin{aligned} 2x_2 + 7x_3 &= 17 \\ 2x_2 + 7(3) &= 17 \\ 2x_2 + 21 &= 17 \\ 2x_2 &= -4 \\ x_2 &= -2 \end{aligned}$$

Now, plugging in x_2 and x_3 into equation 1, we can solve for x_1.

$$\begin{aligned} x_1 + 2x_2 - 3x_3 &= -12 \\ x_1 + 2(-2) - 3(3) &= -12 \\ x_1 - 4 - 9 &= -12 \\ x_1 &= 1 \end{aligned}$$

Exercise 1

Exercise 2. The elementary matrices associated with the elementary row operations in part 1 are

$$E_{21} = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, E_{31} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}, \quad \text{and} \quad E_{32} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix}$$

Multiply these matrices together to get the elimination matrix M.

$$\begin{aligned} M = E_{32}E_{31}E_{21} &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 5 & -2 & 1 \end{pmatrix} \end{aligned}$$

Exercise 2