College of the Redwoods
Mathematics Department
Math 45 — Linear Algebra

Exam #1
Elimination and Matrix Operations

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Last Revision Date: October 4, 2002 Version 1.00

Essay Questions

Read Carefully! *You have the weekend to complete the exam. The exam is due, on my desk, at the beginning of class on Monday.*

This exam is open notes, open book. All work must be done by hand, but you may certainly use a calculator or computer to check your work where appropriate. You must answer all of the exercises on your own. You are not allowed to work in groups on the exam. You are not allowed to enlist the aid of a tutor or friend to help with the exam. You are not allowed to read the exercises in the exam, then seek help on similar questions. Once you open the exam and read the questions, you may not seek any outside help of any kind. From the moment you open the exam, you must do everything by yourself.

Place the solution to each exercise on a separate sheet of paper. On a good sheet of paper, write out (longhand) and sign the following honor pledge.

> I promise that all work found herein is my own. I have received no help from tutors, colleagues, or other teachers. I have honored all of the examination constraints listed in the directions.

Arrange the problems in order, place these examination pages on top of your solutions, then place the honor pledge on top of the examination as a cover sheet. Staple. Good luck!

Exercise 1. Use elimination and back substitution to solve the following system of equations. All computations are to be performed by hand. Show your work.

$$\begin{aligned} x_1 - 2x_2 + 3x_3 &= 12 \\ 2x_1 - 3x_2 + x_3 &= 0 \\ -3x_1 - 4x_2 + 2x_3 &= -21 \end{aligned}$$

Exercise 2. The following matrix represents the reduced row echelon form of an augmented matrix for a system of three equations in unknowns x_1, x_2 and x_3.

$$\begin{pmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & 3 & 4 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

What is the solution of the system?

Exercise 3. Let A be a matrix with three rows.

(a) What 3×3 matrix E adds 5 times row 2 to row 3 and then adds 2 times row 1 to row 2, when it multiplies matrix A?

(b) What 3×3 matrix F subtracts 2 times row 1 from row 2 and then subtracts 5 times row 2 from row 3, when it multiplies matrix A? How is F related to matrix E in the previous question?

Exercise 4. Consider the coefficient matrix

$$A = \begin{pmatrix} 1 & 2 & -1 \\ 2 & 3 & 0 \\ -4 & -5 & -2 \end{pmatrix}.$$

For what triples

$$\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

does the system $A\mathbf{x} = \mathbf{b}$ have a solution?

EXERCISE 5. Suppose that A is a 4×4 matrix such that the sum of the first two columns of matrix A equals the sum of columns 3 and 4.

(a) Find a nonzero solution of $A\mathbf{x}=\mathbf{0}$, where $\mathbf{x}$ is a *column* vector.

(b) Explain why matrix A is noninvertible (singular).

EXERCISE 6. Suppose that B is a 5×5 matrix so that the sum of the first two rows of B is 3 times the sum of rows 3, 4, and 5. Find a nonzero solution of $\mathbf{y}B=\mathbf{0}$, where $\mathbf{y}$ is a *row* vector.

EXERCISE 7. Show how you can express matrix

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix}$$

as the product of two vectors.

EXERCISE 8. Let

$$A=\begin{pmatrix} 1 & 0 \\ -5 & 2 \end{pmatrix}.$$

(a) Find elementary matrices E and F so that $EFA=I$.

(b) Write A^{-1} as a product of elementary matrices.

EXERCISE 9. You learned in class that you can find the inverse of a 2×2 matrix with the formula

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1}=\frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}.$$

Use this formula to find the inverse of the matrix

$$\begin{pmatrix} \cosh x & \sinh x \\ \sinh x & \cosh x \end{pmatrix}.$$

Hint: The hyperbolic cosine and sine are defined as follows:

$$\cosh x=\frac{e^x+e^{-x}}{2} \qquad \text{and} \qquad \sinh x=\frac{e^x-e^{-x}}{2}.$$

EXERCISE 10. Given that

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{pmatrix} A=\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix},$$

find A^{-1}. *Note: This problem is trivial if approached in the correct manner. Look for an elegant solution. Little credit will be given for crunch and grind solutions.*

EXERCISE 11. For what value(s) of c is matrix A singular (not invertible)?

$$A=\begin{pmatrix} 1 & 2 & 3 \\ 1 & 5 & 6 \\ 2 & 6 & c \end{pmatrix}$$

EXERCISE 12. Four by four matrix A consists of four 2×2 blocks.

$$A = \begin{pmatrix} I & B \\ 0 & I \end{pmatrix},$$

where I is the 2×2 identity matrix, 0 is a 2×2 zero matrix (all entries are zeros), and B is any 2×2 invertible matrix.

(a) In block notation, what is the inverse of matrix A?

(b) Craft a 4×4 matrix A that adheres to the pattern described above. Use the technique developed in part (a) to find the inverse of your example.

(c) Use your matrix A from part (b), set up the augmented matrix $[A\ I]$ and use row reduction to place your augmented matrix in the form $[I\ A^{-1}]$. This result should agree with the solution found in part (b)?

Solutions to Exercises

Exercise 1. Take the system

$$
\begin{aligned}
x_1 - 2x_2 + 3x_3 &= 12 \\
2x_1 - 3x_2 + x_3 &= 0 \\
-3x_1 - 4x_2 + 2x_3 &= -21
\end{aligned}
$$

and set up the augmented matrix

$$
\begin{pmatrix} 1 & -2 & 3 & 12 \\ 2 & -3 & 1 & 0 \\ -3 & -4 & 2 & -21 \end{pmatrix}.
$$

Subtract 2 times row 1 from row 2. Subtract -3 times row 1 from row 3.

$$
\begin{pmatrix} 1 & -2 & 3 & 12 \\ 0 & 1 & -5 & -24 \\ 0 & -10 & 11 & 15 \end{pmatrix}
$$

Subtract -10 times row 2 from row 3.

$$
\begin{pmatrix} 1 & -2 & 3 & -12 \\ 0 & 1 & -5 & -24 \\ 9 & 0 & -39 & -225 \end{pmatrix}
$$

This matrix represents the equivalent system

$$
\begin{aligned}
x_1 - 2x_2 + 3x_3 &= 12 && (1) \\
x_2 - 5x_3 &= -25 && (2) \\
-39x_3 &= -225. && (3)
\end{aligned}
$$

We use back substitution to find the solution. Solve (3) for x_3.

$$
\begin{aligned}
-39x_3 &= -225 \\
x_3 &= \frac{225}{39} \\
x_3 &= \frac{75}{13}
\end{aligned}
$$

Substiute $x_3 = 75/13$ in (2) and solve for x_2.

$$
\begin{aligned}
x_2 - 5\left(\frac{75}{13}\right) &= -24 \\
x_2 - \frac{375}{13} &= -\frac{312}{13} \\
x_2 &= \frac{63}{13}
\end{aligned}
$$

Substitute $x_3 = 75/13$ and $x_2 = 63/13$ in (1) and solve for x_1.

$$
\begin{aligned}
x_1 - 2\left(\frac{63}{13}\right) + 3\left(\frac{75}{13}\right) &= 12 \\
x_1 - \frac{126}{13} + \frac{225}{13} &= \frac{156}{13} \\
x_1 + \frac{99}{13} &= \frac{156}{13} \\
x_1 &= \frac{57}{13}
\end{aligned}
$$

Thus, the solution is

$$x_1 = \frac{57}{13}$$
$$x_2 = \frac{63}{13}$$
$$x_3 = \frac{75}{13}$$

Exercise 1

Exercise 2. The augmented matrix

$$\begin{pmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & 3 & 4 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

represents the system

$$x_1 + 2x_2 = 3 \tag{4}$$
$$x_2 + 3x_3 = 4, \tag{5}$$

where x_1 and x_2 are pivot variables and x_3 is a free variable. Solve (4) and (5) for the pivot variables. This gives solution

$$x_1 = 3 - 2x_3$$
$$x_2 = 4 - 3x_3$$
$$x_3 = \text{free}.$$

Exercise 2

Exercise 3(a) The elementary matrix

$$E_{32} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 5 & 1 \end{pmatrix}$$

adds 5 times row 2 to row 3 of matrix A when applied in the order $E_{32}A$. The elementary matrix

$$E_{21} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

adds 2 times row 1 to row 2 of $E_{32}A$ when applied in the order $E_{21}E_{32}A$. Hence, the matrix we seek is

$$E = E_{21}E_{32} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 5 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 5 & 1 \end{pmatrix}.$$

□

Exercise 3(b) The elementary matrix

$$F_{21} = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

subtracts 2 times row 1 from row 2 of matrix A when applied in the order $F_{21}A$. The elementary matrix

$$F_{32} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -5 & 1 \end{pmatrix}$$

subtracts 5 times row 2 of $F_{21}A$ when applied in the order $F_{32}F_{21}A$. Hence, the matrix we seek is

$$F = F_{32}F_{21} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -5 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 10 & -5 & 1 \end{pmatrix}$$

Because the marix F reverses the steps of matrix E in inverse order, it must be the case that F is the inverse of matrix E. This is easily checked. Note that

$$F_{32}^{-1} = E_{32} \qquad \text{and} \qquad F_{21}^{-1} = E_{21}.$$

Hence, these relations provide

$$\begin{aligned} FE &= F_{32}F_{21}E_{21}E_{32} \\ &= F_{32}F_{21}F_{21}^{-1}F_{32}^{-1} \\ &= F_{32}IF_{32}^{-1} \\ &= F_{32}F_{32}^{-1} \\ &= I. \end{aligned}$$

Similarly, it can be shown the $EF = I$. Hence, F and E are inverses of one another.

□

Exercise 4. Set up the augmented matrix

$$M = \begin{pmatrix} A & \mathbf{b} \end{pmatrix} = \begin{pmatrix} 1 & 2 & -1 & b_1 \\ 2 & 3 & 0 & b_2 \\ -4 & -5 & -2 & b_3 \end{pmatrix}.$$

Subtract 2 times row 1 from row 2. Subtract -4 times row 1 from row 3.

$$\begin{pmatrix} 1 & 2 & -1 & b_1 \\ 0 & -1 & 2 & b_2 - 2b_1 \\ 0 & 3 & -6 & b_3 + 4b_1 \end{pmatrix}$$

Subtract -3 times row 2 from row 3.

$$\begin{pmatrix} 1 & 2 & -1 & b_1 \\ 0 & -1 & 2 & -2b_1 + b_2 \\ 0 & 0 & 0 & -2b_1 + 3b_2 + b_3 \end{pmatrix}$$

Because the last equation of the system represented by this matrix is

$$0x_1 + 0x_2 + 0x_3 = -2b_1 + 3b_2 + b_3,$$

the system will have solutions if and only if

$$-2b_1 + 3b_2 + b_3 = 0.$$

Exercise 4

Exercise 5(a) Matrix A has the form

$$A = \begin{pmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 & \mathbf{a}_4 \end{pmatrix},$$

where we're given that

$$\mathbf{a}_1 + \mathbf{a}_2 = \mathbf{a}_3 + \mathbf{a}_4,$$

or, equivalently,

$$1\mathbf{a}_1 + 1\mathbf{a}_2 - 1\mathbf{a}_3 - 1\mathbf{a}_4 = \mathbf{0}.$$

Clearly, $\mathbf{x} = (1, 1, -1, -1)^T$ is a solution, as

$$\begin{aligned} A\mathbf{x} &= \begin{pmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 & \mathbf{a}_4 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ -1 \\ -1 \end{pmatrix} \\ &= 1\mathbf{a}_1 + 1\mathbf{a}_2 - 1\mathbf{a}_3 - 1\mathbf{a}_4 \\ &= \mathbf{0}. \end{aligned}$$

□

Exercise 5(b) Suppose, for purposes of contradiction, that A is invertible. Then A^{-1} exists and

$$\begin{aligned} A\mathbf{x} &= \mathbf{0} \\ A^{-1}(A\mathbf{x}) &= A^{-1}\mathbf{0} \\ (A^{-1}A)\mathbf{x} &= \mathbf{0} \\ I\mathbf{x} &= \mathbf{0} \\ \mathbf{x} &= \mathbf{0}. \end{aligned}$$

This says that the only solution of $A\mathbf{x} = \mathbf{0}$ is $\mathbf{x} = \mathbf{0}$, which contradicts the finding in part (a). Thus, A is not invertible.

□

Matrix B has the form

$$B = \begin{pmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \end{pmatrix},$$

where

$$B_1 + B_2 = 3(B_3 + B_4 + B_5),$$

or, equivalently,

$$1B_1 + 1B_2 - 3B_3 - 3B_4 - 3B_5 = 0.$$

Clearly, $\mathbf{y} = (1, 1, -3, -3, -3)$ is a solution as

$$\begin{aligned} \mathbf{y}B &= \begin{pmatrix} 1 & 1 & -3 & -3 & -3 \end{pmatrix} \begin{pmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \end{pmatrix} \\ &= 1B_1 + 1B_2 - 3B_3 - 3B_4 - 3B_5 \\ &= \mathbf{0}. \end{aligned}$$

□

Exercise 7. Using block multiplication,

$$\begin{aligned} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \end{pmatrix} &= \begin{pmatrix} 1\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} & 2\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} & 3\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} & 4\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} & 5\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \end{pmatrix} \\ &= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix}. \end{aligned}$$

Exercise 7

Exercise 8(a) Let

$$A = \begin{pmatrix} 1 & 0 \\ -5 & 2 \end{pmatrix}.$$

Subtract -5 times row 1 from row 2.

$$FA = \begin{pmatrix} 1 & 0 \\ 5 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -5 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$$

Now, multiply the second row of FA by $1/2$.

$$EFA = \begin{pmatrix} 1 & 0 \\ 0 & 1/2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I.$$

□

Exercise 8(b) Hence,

$$A^{-1} = EF = \begin{pmatrix} 1 & 0 \\ 0 & 1/2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 5 & 1 \end{pmatrix}.$$

□

Exercise 9. We will use the formula

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

to invert

$$\begin{pmatrix} \cosh x & \sinh x \\ \sinh x & \cosh x \end{pmatrix}^{-1} = \frac{1}{\cosh^2 x - \sinh^2 x} \begin{pmatrix} \cosh x & -\sinh x \\ -\sinh x & \cosh x \end{pmatrix}.$$

However,

$$\begin{aligned} \cosh^2 x - \sinh^2 x &= \left(\frac{e^x + e^{-x}}{2}\right)^2 - \left(\frac{e^x - e^{-x}}{2}\right)^2 \\ &= \frac{e^{2x} + 2 + e^{-2x}}{4} - \frac{e^{2x} - 2 + e^{-2x}}{4} \\ &= \frac{4}{4} \\ &= 1. \end{aligned}$$

Hence,

$$\begin{pmatrix} \cosh x & \sinh x \\ \sinh x & \cosh x \end{pmatrix}^{-1} = \begin{pmatrix} \cosh x & -\sinh x \\ -\sinh x & \cosh x \end{pmatrix}.$$

Exercise 9

Exercise 10. We're given that

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{pmatrix} A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}. \tag{6}$$

To find an inverse, we need the right-hand side of equation (6) to equal the identity matrix I. First, swap rows 1 and 3.

$$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{pmatrix} A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 8 \\ 2 & 5 & 3 \\ 1 & 2 & 3 \end{pmatrix} A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Next, sqap rows 2 and 3.

$$\begin{pmatrix}1 & 0 & 0\\ 0 & 0 & 1\\ 0 & 1 & 0\end{pmatrix}\begin{pmatrix}1 & 0 & 8\\ 2 & 5 & 3\\ 1 & 2 & 3\end{pmatrix}A = \begin{pmatrix}1 & 0 & 0\\ 0 & 0 & 1\\ 0 & 1 & 0\end{pmatrix}\begin{pmatrix}1 & 0 & 0\\ 0 & 0 & 1\\ 0 & 1 & 0\end{pmatrix}$$

$$\begin{pmatrix}1 & 0 & 8\\ 1 & 2 & 3\\ 2 & 5 & 3\end{pmatrix}A = \begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}$$

Equivalently,

$$\begin{pmatrix}1 & 0 & 8\\ 1 & 2 & 3\\ 2 & 5 & 3\end{pmatrix}A = I.$$

Thus,

$$A^{-1} = \begin{pmatrix}1 & 0 & 8\\ 1 & 2 & 3\\ 2 & 5 & 3\end{pmatrix}.$$

Exercise 10

Exercise 11. We attempt to tow reduce matrix

$$A = \begin{pmatrix}1 & 2 & 3\\ 1 & 5 & 6\\ 2 & 6 & c\end{pmatrix}$$

to the identity. Subtract 1 times row 1 from row 2. Subtract 2 times row 1 from row 3.

$$\begin{pmatrix}1 & 2 & 3\\ 0 & 3 & 3\\ 0 & 2 & c-6\end{pmatrix}$$

Subtract 2/3 times row 2 from row 3.

$$\begin{pmatrix}1 & 2 & 3\\ 0 & 3 & 3\\ 0 & 0 & c-8\end{pmatrix}$$

If $c = 8$, then the entry in row 3, column 3 will be zero and there is no hope of continuing the reduction to the identity matrix. Hence, if $c = 8$, matrix A is not invertible.

Exercise 11

Exercise 12(a) To find the inverse of matrix

$$A = \begin{pmatrix}I & B\\ 0 & I\end{pmatrix},$$

we first craft the augmented matrix

$$\begin{pmatrix}I & B & I & 0\\ 0 & I & 0 & I\end{pmatrix}.$$

We must reduce A to the identity matrix. Subtract B times row 2 from row 1.

$$\begin{pmatrix}I & 0 & I & -B\\ 0 & I & 0 & I\end{pmatrix}.$$

Hence,

$$A^{-1} = \begin{pmatrix}I & -B\\ 0 & I\end{pmatrix}.$$

□

Exercise 12(b) Consider

$$A = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

which in block form, has the form

$$A = \begin{pmatrix} I & B \\ 0 & I \end{pmatrix}.$$

By part (a),

$$\begin{aligned} A^{-1} &= \begin{pmatrix} I & -B \\ 0 & I \end{pmatrix} \\ &= \begin{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} & \begin{pmatrix} -1 & 0 \\ -2 & -1 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \end{aligned}$$

□

Exercise 12(c) Set up

$$\begin{pmatrix} A & I \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

Subtract 1 times row 4 from row 2.

$$\begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

Subtract 2 times row 3 from row 2. Subtract 1 times row 3 from row 1.

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & -2 & -1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

Hence,

$$A^{-1} = \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

which agrees with the result found in part (b).

□