Velocity and Acceleration Vectors in Matlab

Math 50C — Multivariable Calculus
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Abstract. If a particle moves along a path with constant speed, then its acceleration and velocity vectors will be orthogonal at each position along the path. Matlab is used to provide a clear visualization of this fact.

Prerequisites. You must understand how to find the position, velocity, and acceleration of a particle as it travels through space. Some familiarity with plotting in Matlab. You'll also need a rudimentary understanding of Matlab's element-wise operators (.∗, ./, .ˆ).

Position, Velocity, and Acceleration

If the position of a particle in R^3 is given by

$$\mathbf{r}(t) = \cos t\mathbf{i} + \sin t\mathbf{j} + t\mathbf{k} \tag{1}$$

then taking the first and second derivatives will produce the velocity and acceleration, respectively.

$$\mathbf{v}(t) = -\sin t\mathbf{i} + \cos t\mathbf{j} + \mathbf{k} \tag{2}$$
$$\mathbf{a}(t) = -\cos t\mathbf{i} - \sin t\mathbf{j} + 0\mathbf{k} \tag{3}$$

Constant Speed

Suppose that the speed of a particle as it travels through space is constant. The velocity need not be constant, only the speed. That is, the direction of the velocity may be changing, but the magnitude of the velocity (the length of the velocity vector) remains constant. If the magnitude of the velocity is constant, then certainly the square of the magnitude of the velocity is also constant and you may write

$$\|\mathbf{v}\|^2 = C$$

Use the fact that $\|\mathbf{v}\|^2 = \mathbf{v} \cdot \mathbf{v}$, then differentiate both sides of the resulting equation with respect to t.

$$\begin{aligned}
\mathbf{v} \cdot \mathbf{v} &= C \\
\frac{d}{dt}(\mathbf{v} \cdot \mathbf{v}) &= \frac{d}{dt}C \\
\frac{d\mathbf{v}}{dt} \cdot \mathbf{v} + \mathbf{v} \cdot \frac{d\mathbf{v}}{dt} &= 0 \\
2\left(\mathbf{v} \cdot \frac{d\mathbf{v}}{dt}\right) &= 0
\end{aligned}$$

$$\mathbf{v}\cdot\frac{d\mathbf{v}}{dt} = 0 \tag{4}$$

Of course, acceleration is the derivative of the velocity with respect to time, so equation (4) can be written as

$$\mathbf{v}\cdot\mathbf{a}=0 \tag{5}$$

Thus, if the speed (magnitude of the velocity) is constant, then velocity and acceleration vectors must be orthogonal (because their dot product is zero).

Exercise 1 (Homework Exercise #1) *If a particle's position in space is given by equation (1), show that the particle has constant speed. (*Hint: Use equation (2) to find the magnitude of the velocity)*.*

Exercise 2 (Homework Exercise #2) *If a particle's position in space is given by equation (1), show that the velocity and acceleration vectors are orthogonal at each position along the particle's path. (*Hint: Take the dot product of the vectors in (2) and (3).*)*

Visualization in Matlab

You will now create a Matlab M-file that will plot the path of the particle and attach vectors representing velocity and acceleration at selected points along the path of the particle. Start by plotting two turns of the helix defined by equation (1).

```
>> close all
>> t=linspace(0,4*pi);
>> x=cos(t);y=sin(t);z=t;
>> plot3(x,y,z)
```

- Hold the plot for additional plotting.

```
>> hold on
```

- Choose several points on the particle's path at which to plot the velocity and acceleration vectors.

```
>> tt=linspace(0,4*pi,9);
>> xx=cos(tt);yy=sin(tt);zz=tt;
```

- Calculate the velocity and acceleration vectors at each of these selected points.

```
>> vx=-sin(tt);vy=cos(tt);vz=ones(size(tt));
>> ax=-cos(tt);ay=-sin(tt);az=zeros(size(tt));
```

- Use Matlab's `quiver3` command to plot the vectors.

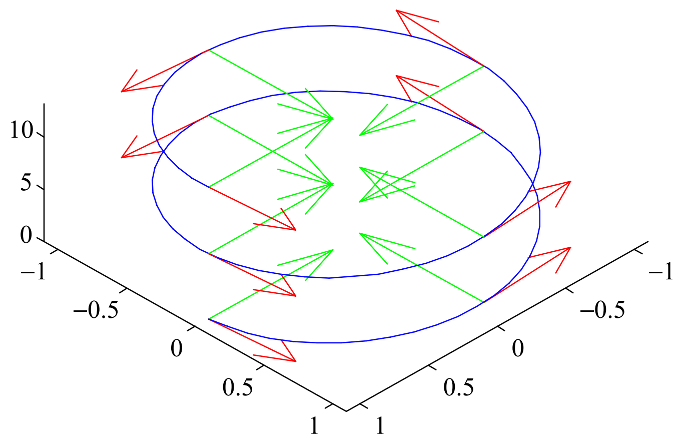

Figure 1: The velocity and acceleration vectors are orthogonal.

```
quiver3(xx,yy,zz,vx,vy,vz,'r')
quiver3(xx,yy,zz,ax,ay,az,'g')
```

- Use Matlab's `rotate3d` command to move the figure into a position similar to that in Figure 1. Note how the velocity and acceleration vectors are orthogonal at each point of the particle's path. You might also want to experiment with `axis tight` and `axis equal`.

Exercise 3 (Homework Exercise #3) *Obtain a printout of the image in Figure 1. Label each axis and include a title containing the equation of the particle's path.*

Exercise 4 (Extra Credit) *If the speed is not constant, then the velocity and acceleration vectors are not necessarily orthogonal at each point on the particle's path. Find an example of this behavior and demonstrate it visually using the Matlab procedures outlined in this activity.*