College of the Redwoods
Mathematics Department

Exam #2
Review Questions

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Instructions. *Please place the solution of each of the following questions on your own paper. You must show all supporting work to receive credit for your solution. If you are asked to write a response, grammar, spelling, punctuation, and style are important.*

Exercise 1. The English SAT scores follow a normal distribution $N(510, 80)$ while English ACT scores follow a normal distribution $N(20, 6)$. Liz scores 600 on the SAT and Tim scores 22 on the ACT.

(a) What is Liz's standard score?

(b) What is Tim's standard score?

(c) Who has the higher score relative to the corresponding distribution of scores?

Exercise 2. Sketch the standard normal curve, paying special attention to the scale on the horizontal axis and the location of the points of inflection of the standard normal curve.

Exercise 3. In each of the following exercises, sketch a picture of the area under the standard normal density curve, then use your calculator to find the corresponding area.

(a) The area to the right of $z = 0.84$.

(b) The area to the left of $z = 2.12$.

(c) The area between $z = -1.12$ and $z = 2.21$.

Exercise 4. Repeate Exercise 3 without the use of a calculator. Use the tables on pages 942-943 in your text.

Exercise 5. Sketch a picture of the following descriptions regarding the standard normal density, then use your calculator to find the corresponding z-values.

(a) The z-value corresponding to the 90th percentile.

(b) The z-value corresponding to the first quartile Q_1.

(c) The z-values (symmetric about 0) whose area between is 0.60.

Exercise 6. Repeat Exercise 5 without the use of a calculator. Use the tables on pages 942-943 of your text.

Exercise 7. Scores on an assessment test in mathematics are approximately normal with a mean of 50 and a standard deviation of 5. Sketch the distribution of scores. Label your axis appropriately and carefully locate the points of inflection of the normal curve.

Exercise 8. The lifetime of Gadget batteries is normally distributed with a mean of 200 hours and a standard deviation 0f 25 hours.

(a) What percentage of the batteries have a lifetime of at least 180 hours?

(b) What percentage of the batteries have a lifetime of at most 212 hours?

(c) Batteries is the top 5% of this distribution have a minimum life of how many hours?

EXERCISE 9. Machine A makes pipe whose diameters are normally distributed with a mean of 100 mm and a standard deviation of 4 mm. Machine B makes pipe whose diameters are normally distributed with a mean of 104 mm and a standard deviation of 3 mm. You have a box of pipe which you believe to be from machine A, but you are not sure. You decide to test the hyposthesis:

$$H_0: \text{ The pipe is from machine A}$$
$$H_1: \text{ The pipe is from machine B}$$

Decision Rule: You decide to reject the null hypothesis if the diameter of a selected pipe is greater than 106 mm.

(a) What is the probability of making a Type I error?

(b) What is the probability of making a Type II error?

EXERCISE 10. In Exercise 9, suppose that a pipe having diameter 103 mm is selected. Calculate the p-value for this selection.

EXERCISE 11. A company making potato chips claims that the average weight of a bag of their potato chips is normally distributed with a mean weight of 16 ounces with a standard deviation of 2.3 ounces. A customer complaint group claims that the mean weight is significantly lower and decides to test the claim of the company.

(a) Set up the null and alternative hypothesis for the test.

(b) What is the direction of extreme?

(c) A potato chip bag is selected at random and weighed. If its weight is 14 ounces, what is the corresponding p-value?

EXERCISE 12. Suppose that X is uniformly distributed on the interval $(0, 10)$.

(a) Sketch the density function for the distribution of X.

(b) What is the mean of the distribution?

(c) Shade the region representing the probability that X is at least 7. What is the probability that X is at least 7?

(d) Shade the region representing the probability that X is at most 3. What is the probability that X is at most 3?

EXERCISE 13. The Fortuna Post Office states that the length of time a customer must stand in line is uniformly distributed between 0 and 10 minutes.

(a) Sketch the distribution of the time a customer waits in line.

(b) What is the mean wait in line?

(c) Customers have been complaining that they must actually weight longer than the reported mean. Set up the null and alternative hypothesis for this test.

(d) A customer is selected at random and his time standing in line is found to be 8 minutes. Calculate the corresponding p-value.

(e) What decision would you make at the 10% significance level?

Exercise 14. The manager of a large movie complex reports that the wait in line for a new release is uniformly distributed between 10 and 30 minutes. A customer complaint group claims that the wait in line is uniformly distributed between 20 and 40 minutes. A test is setup up with the following null and alternative hypothesis.

$$H_0 : \text{The manager is correct}$$
$$H_1 : \text{The customer complaint group is correct}$$

A patron is selected at random and his time in line is recorded. Decision Rule: Reject H_0 if the patron's wait is line is longer than 25 minutes.

(a) Calculate the probability of a Type I error.

(b) Calculate the probability of a Type II error.

Exercise 15. A four-side die in the shape of a tetrahedron is "fair." That is, any one of the four sides has a equal chance of being selected. The number 1 is on one side of the die, 2 on another side, 3 on another side, and a 4 is on the remaining side. Suppose that the die is rolled and comes to rest on a table top. The die is lifted and the side whose face is on the table top is inspected and the number on this side is noted.

(a) What is the sample space for this random process?

(b) Let A be the event that a 1 is thrown on the first toss. List the outcomes from the sample space that comprise the event A. What is $P(A)$, the probability of the event A occurring?

(c) Let B be the event that a 1 is thrown on the second toss. List the outcomes from the sample space that comprise the event B. What is $P(B)$, the probability of the event B occurring?

(d) Are the events A and B mutually exclusive (disjoint)? Why?

(e) Are the events A and B independent? Why? Yes.

$$P(A \cap B) = 1/16 \qquad \text{and} \qquad P(A)P(B) = (1/4)(1/4) = 1/16.$$

Exercise 16. A standard six-sided die has the numbers 1 through six on its faces. That is, one side shows the number 1, another side the number 2, and so on. The die is "fair," meaning that any one of the numbers 1 through 6 has an equal chance of being selected. The die is rolled and allowed to come to rest on a table. The number on the top face of the die is noted. The procedure is repeated. That's two separate rolls of the die.

(a) List all the outcomes in the sample space for this random process.

(b) Let A be the event that a 6 is rolled on the first die. What is $P(A)$?

(c) Let B be the event that a 5 is rolled on the second die. What is $P(B)$?

(d) What is $P(A \cap B)$?

(e) Are the events A and B independent? Why? Yes.

$$P(A \cap B) = 1/36 \qquad \text{and} \qquad P(A)P(B) = (1/6)(1/6) = 1/36.$$

(f) Are the events A and B mutually exclusive? Why? No. They have $(6, 5)$ in common.

Exercise 17. In Exercise 16, two fair die are tossed. Answer each of the following questions.

(a) What is the probability that the sum of the faces is seven?

(b) What is the probability that the sum of the faces is at most 8?

(c) What is the probability that the sum of the faces is at least 8?

Exercise 18. Refer again to the experiment and its sample space in Exercise 16.

(a) What is the probability that the sum of the two die is 7, knowing that the second die shows a 3?

(b) What is the probability that the sum of the two die is at most 5 knowing that the second die shows a 3?

Exercise 19. Let A and B be events in a sample space such that $P(A) = 0.4$, $P(B) = 0.6$ and $P(A \cap B) = 0.2$. Draw a Venn diagram with the proportions probabilities recorded. Answer each of the following questions.

(a) What is $P(A^C)$?

(b) What is $P(A \cup B)$?

(c) What is the $P(A \cap B^C)$?

(d) What is $P(A|B)$?

(e) What is $P(B|A)$?

Exercise 20. Suppose that $P(A) = P(B) = 0.4$ and $P(A|B) = 0.3$. What is $P(A \cup B)$?

Exercise 21. In a certain community, males and females eligible to vote are classified as Democrat, Republican, or Other as shown in the following table.

	Democrat	Republican	Other
Males	100	120	30
Females	90	130	20

A person is selected at random from the community.

(a) What is the probability that the person is male? female?

(b) What is the probability that the person is Democrat? Republican? Other?

(c) What is the probability that the person is either female or Democrat?

(d) What is the probability that the person is male and Republican?

(e) What is the probability that the person is male, knowing that the person is a Democrat?

(f) What is the probability that the person is female, knowing that the person is Republican?

EXERCISE 22. Let X be the number of books in a student backpack for a certain class of students. Consider the following probability distribution of X, which assumes that there are no more than 5 books in any student backpack.

x	0	1	2	3	4	5
p	0.05	0.10	0.40	0.30	0.10	

Answer each of the following.

(a) What is the probability that a randomly selected student has 5 books in her backpack?

(b) What is the probability that a randomly selected student has at most 3 books in his backpack?

(c) What is the probability that a randomly selected student has at least 2 books in her backpack?

EXERCISE 23. Refer to Exercise 22 and answer each of the follwowing.

(a) Sketch the mass density function of the random variable X.

(b) Find $E(X)$, the "expectation" of the random variable X. Mark this value at the appropriate position of your mass density plot in part (a).

(c) What is $\text{Var}(X)$, the "variance" of the random variable X? What is the standard deviation of X?

EXERCISE 24. Evaluate each of the following *without* the use of a calculator.

(a) $5!$ (b) ${}_5P_3$ (c) ${}_5C_3$ (d) $\binom{7}{2}$

EXERCISE 25. Evaluate the following.

$$\binom{7}{2}(1/4)^2(3/4)^5$$

EXERCISE 26. A fair coin is tossed five times. Define a random variable X by saying that X equals the number of heads obtained in the five independent tossing of the coin. Complete each of the following tasks.

(a) Note that the number of heads in five tosses of the fair coin could equal 0, 1, 2, 3, 4, or 5. That is, the random variable X is a discrete (binomial) random variable with possible values 0, 1, 2, 3, 4, or 5. Complete the following table of probabilities for each possible value of the random variable.

x	p
0	
1	
2	
3	
4	
5	

(b) On graph paper, sketch the mass density function using the data from the table in part (a).

(c) Add another column to your table in part (a) and complete the missing entries to calculate the expectation of the random variable X via the formula $\mu = E(X) = \sum xp$.

x	p	xp
0		
1		
2		
3		
4		
5		
		$\sum xp =$

Now, as a check of your work, use the formula for finding the expectation of a binomial random variable,

$$\mu = E(X) = np,$$

to calculate the expectation of the binomial random variable X. Make sure that it matches the result found in your table before continuing. Finally, place this result on the mass density function in part (b) in the appropriate location.

(d) Add another column to your table in part (c) and complete the missing entries to calculate the variance of the random variable X via the formula $\sigma^2 = \text{Var}(X) = \sum x^2p - \mu^2$.

x	p	xp	x^2p
0			
1			
2			
3			
4			
5			
		$\sum xp =$	$\sum x^2p =$

Now, as a check of your work, use the formula for finding the variance of a binomial random variable,

$$\sigma^2 = \text{Var}(X) = npq,$$

to calculate the variance of the binomial random variable X. Make sure that it matches the result found in your table before continuing.

EXERCISE 27. A fair (the probability of heads is 1/2 and the probability of tails is 1/2) coin is tossed ten times.

(a) What is the probability of getting 8 heads?

(b) What is the probability of getting at most 7 heads?

(c) What is the probability of getting at least 3 heads?

Exercise 28. Repeat the calculations in Exercise 27 if the coin is unfair and the probability of getting a head on any one toss is 0.6.

Solutions to Exercises

Exercise 1(a) 1.1250

□

Exercise 1(b) 0.3333

□

Exercise 1(c) Liz.

□

Exercise 2. **Exercise 3(a)** 0.2005

□

Exercise 3(b) 0.9830

□

Exercise 3(c) 0.8551

□

Exercise 4. **Exercise 5(a)** 1.2816

□

Exercise 5(b) −0.6745

□

Exercise 5(c) −0.8416

□

Exercise 6. **Exercise 7.** **Exercise 8(a)** 0.7881

□

Exercise 8(b) 0.6843

□

Exercise 8(c) 241.12

□

Exercise 9(a) 0.0668

□

Exercise 9(b) 0.7475

□

Exercise 10. 0.2266

Exercise 10

Exercise 11(a) H_0: $\mu = 16$, H_1: $\mu < 16$

□

Exercise 11(b) left

□

Exercise 11(c) 0.1923

□

Exercise 12(a) **Exercise 12(b)** 5

□

Exercise 12(c) 3/10

□

Exercise 12(d) 3/10

□

Exercise 13(a) **Exercise 13(b)** 5

□

Exercise 13(c) H_0; $\mu = 5$, H_1: $\mu > 5$

□

Exercise 13(d) 2/10

□

Exercise 13(e) Accept the null.

□

Exercise 14(a) 0.25

□

Exercise 14(b) 0.25

□

Exercise 15(a)

$$S = \begin{Bmatrix} (1,1), & (1,2), & (1,3), & (1,4) \\ (2,1), & (2,2), & (2,3), & (2,4) \\ (3,1), & (3,2), & (3,3), & (3,4) \\ (4,1), & (4,2), & (4,3), & (4,4) \end{Bmatrix}$$

□

Exercise 15(b)

$$A = \big\{(1,1),(1,2),(1,3),(1,4)\big\}$$

Thus, $P(A) = 1/4$.

□

Exercise 15(c)

$$B = \big\{(1,1),(2,1),(3,1),(4,1)\big\}$$

Thus, $P(B) = 1/4$.

□

Exercise 15(d) No. they have $(1,1)$ in common.

□

Exercise 15(e) **Exercise 16(a)**

$$S = \left\{ \begin{matrix} (1,1), & (1,2), & (1,3), & (1,4), & (1,5), & (1,6) \\ (2,1), & (2,2), & (2,3), & (2,4), & (2,5), & (2,6) \\ (3,1), & (3,2), & (3,3), & (3,4), & (3,5), & (3,6) \\ (4,1), & (4,2), & (4,3), & (4,4), & (4,5), & (4,6) \\ (5,1), & (5,2), & (5,3), & (5,4), & (5,5), & (5,6) \\ (6,1), & (6,2), & (6,3), & (6,4), & (6,5), & (6,6) \end{matrix} \right\}$$

□

Exercise 16(b)

$$A = \{(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$$

Thus, $P(A) = 1/6$.

□

Exercise 16(c)

$$B = \{(1,5),(2,5),(3,5),(4,5),(5,5),(6,5)\}$$

Thus, $P(B) = 1/6$.

□

Exercise 16(d)

$$A \cap B = \{(6,5)\}$$

Thus, $P(A \cap B) = 1/36$.

□

Exercise 16(e) **Exercise 16(f)** **Exercise 17(a)** 1/6

□

Exercise 17(b) 26/36

□

Exercise 17(c) 15/36

□

Exercise 18(a) 1/6

□

Exercise 18(b) 1/3

□

Exercise 19(a) 0.6

□

Exercise 19(b) 0.8

□

Exercise 19(c) 0.2

□

Exercise 19(d) 1/3

□

Exercise 19(e) 1/2

□

Exercise 20. **Exercise 21(a)** 250/490, 240/490

□

Exercise 21(b) 190/490, 250/490, 50/490

□

Exercise 21(c) 340/490

□

Exercise 21(d) 120/490

□

Exercise 21(e) 100/190

□

Exercise 21(f) 130/250

□

Exercise 22(a) 0.05

□

Exercise 22(b) 0.85

□

Exercise 22(c) 0.85

□

Exercise 23(a) **Exercise 23(b)** 2.5

□

Exercise 23(c) 1, 1

□

Exercise 24(a) 120

□

Exercise 24(b) 60

□

Exercise 24(c) 10

□

Exercise 24(d) 21

□

Exercise 25. 0.3114

Exercise 25

Exercise 26(a)

x	p
0	0.03125
1	0.15625
2	0.31250
3	0.31250
4	0.15625
5	0.03125

□

Exercise 26(b) **Exercise 26(c)**

x	p	xp
0	0.03125	0.00000
1	0.15625	0.15625
2	0.31250	0.62500
3	0.31250	0.93750
4	0.15625	0.62500
5	0.03125	0.15625
		$\sum xp = 2.5$

Thus,

$$\mu = E(X) = \sum xp = 2.5.$$

Now, using the formula for the mean of a binomial random variable,

$$\mu = E(X) = np = (5)(1/2) = 2.5.$$

□

Exercise 26(d)

x	p	xp	x^2p
0	0.03125	0.00000	0.00000
1	0.15625	0.15625	0.15625
2	0.31250	0.62500	1.25000
3	0.31250	0.93750	2.81250
4	0.15625	0.62500	2.50000
5	0.03125	0.15625	0.78125
		$\sum xp = 2.5$	$\sum x^2p = 7.5$

Thus,

$$\sigma^2 = \text{Var}(X) = \sum x^2p - \mu^2 = 7.5 - (2.5)^2 = 1.25.$$

Now, using the formula for the variance of a binomial random variable,

$$\sigma^2 = \text{Var}(X) = npq = 5(1/2)(1/2) = 1.25$$

□

Exercise 27(a) 0.0439

□

Exercise 27(b) 0.9453

□

Exercise 27(c) 0.9453

□

Exercise 28. $P(X = 8) = 0.1209$, $P(X \leq 7) = 0.8327$, $P(X \geq 3) = 1 - P(X \leq 2) = 0.9877$

Exercise 28