10pts

1. $p(x) = 2x^3 - 9x^2 - 24x + 10$

$\overline{}$ / 60

$p'(x) = 6x^2 - 18x - 24$

$p'(x) = 6(x^2 - 3x - 4)$

$p'(x) = 6(x-4)(x+1)$

Critical Values: 4, −1

p	inc		dec		inc
p'	$+$	-1	$-$	4	$+$
		↑ local max		↑ local min	

$p(-1) = 2(-1)^3 - 9(-1)^2 - 24(-1) + 10 = 23$ (local max)

$p(4) = 2(4)^3 - 9(4)^2 - 24(4) + 10 = -102$ (local min)

Second Derivative:

$p''(x) = 12x - 18$

$0 = 12x - 18$

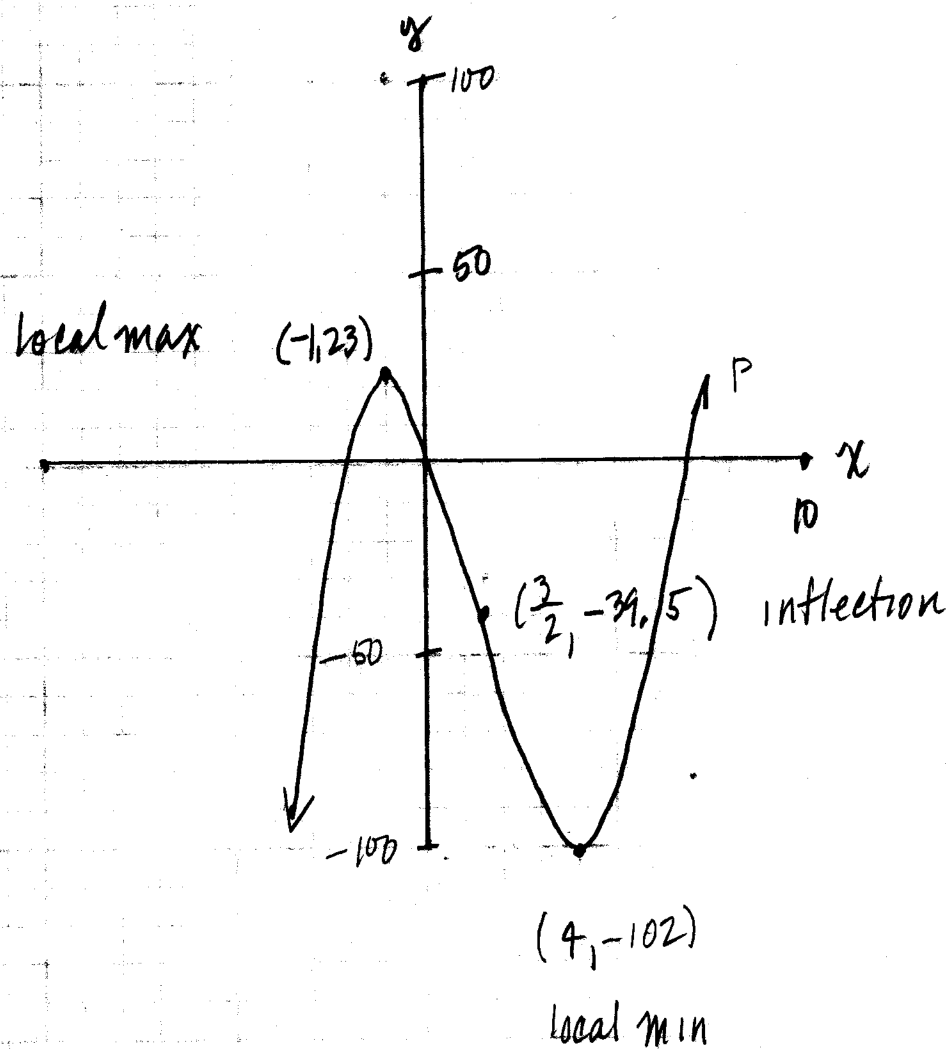
$12x = 18$

$x = \frac{18}{12}$

$x = \frac{3}{2}$

p	down		up
p''	$-$	$\frac{3}{2}$	$+$
		↑ infl. pt.	

$p(\frac{3}{2}) = 2(\frac{3}{2})^3 - 9(\frac{3}{2})^2 - 24(\frac{3}{2}) + 10 = -39.5$ (inflection point)



5pts

② $\displaystyle \lim_{x \to 0} \frac{1-\cos x}{x^2} = \lim_{x \to 0} \frac{\frac{d}{dx}(1-\cos x)}{\frac{d}{dx} x^2}$

$\displaystyle = \lim_{x \to 0} \frac{\sin x}{2x}$

$\displaystyle = \lim_{x \to 0} \frac{\frac{d}{dx} \sin x}{\frac{d}{dx} 2x}$

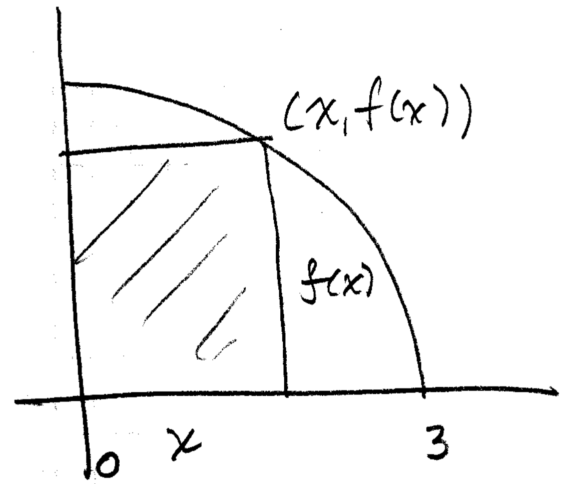
$\displaystyle = \lim_{x \to 0} \frac{\cos x}{2}$

$= \frac{1}{2}$

(3) $f(x)=\sqrt{9-x^2}$

The area of the rectangle is

$$A = x f(x)$$
$$A = x\sqrt{9-x^2}$$



The empirical domain is $[0,3]$. A is continuous on $[0,3]$ so it must have a max, which will occur at an endpoint or critical value.

$$\frac{dA}{dx} = x\left[\tfrac{1}{2}(9-x^2)^{-1/2}(-2x)\right] + 1\cdot(9-x^2)^{1/2}$$
$$= -x^2(9-x^2)^{-1/2} + (9-x^2)^{1/2}$$
$$= (9-x^2)^{-1/2}\left[-x^2+(9-x^2)\right]$$
$$= (9-x^2)^{-1/2}\left[9-2x^2\right]$$
$$= \frac{9-2x^2}{\sqrt{9-x^2}}$$

The numerator is zero at;

$$9-2x^2=0$$
$$x^2=\tfrac{9}{2}$$
$$x=\pm\, 3/\sqrt{2}$$

Only $3/\sqrt{2}$ is in domain $[0,3]$.

The fraction is undefined when

$$9-x^2 \le 0$$

$$x^2 \ge 9$$

$$\sqrt{x^2} \ge \sqrt{9}$$

$$|x| \ge 3$$

Undefined on $\{x : x \le -3 \text{ or } x \ge 3\}$. Only 3 is in the domain. $[0,3]$.

Critical values are $3/\sqrt{2}$ and 3.

Put endpoints and critical values in a table.

$$A\left(\tfrac{3}{\sqrt{2}}\right) = \frac{3}{\sqrt{2}}\sqrt{9-\left(\tfrac{3}{\sqrt{2}}\right)^2}$$

$$= \frac{3}{\sqrt{2}}\sqrt{9-\tfrac{9}{2}}$$

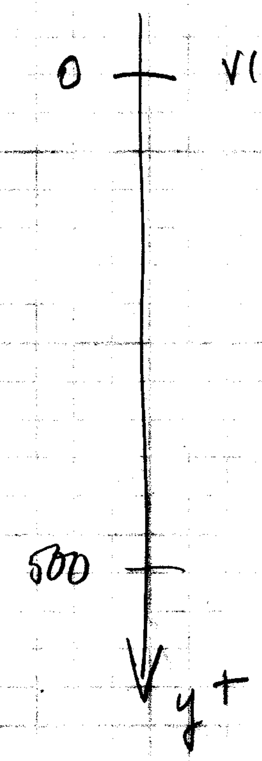
$$= \frac{3}{\sqrt{2}}\sqrt{\tfrac{9}{2}}$$

$$= \frac{3}{\sqrt{2}}\cdot\frac{3}{\sqrt{2}}$$

$$= 9/2$$

x	$A(x)$
0	0
$3/\sqrt{2}$	$9/2$ ← Max.
3	0

~~10pts~~ 4



$v(0)=0,\ y(0)=0$

$$a = 9.8$$

$$\frac{dv}{dt} = 9.8$$

Antidifferentiate:

$$v = 9.8t + C$$

Note that

$$v(0) = 0$$
$$9.8(0) + C = 0$$
$$C = 0$$

Therefore, $v = 9.8t$.

$$\boxed{v = 9.8t}$$ 1

$$\frac{dy}{dt} = 9.8t$$

$$y = 4.9t^2 + D$$

But $y(0)=0$, so

$$y(0) = 0$$
$$4.9(0)^2 + D = 0$$
$$D = 0$$

Thus,

$$\boxed{y = 4.9t^2} \quad (2).$$

To find time required to hit ground, let $y = 500$ m in (2),

$$500 = 4.9t^2$$

$$t^2 = \frac{500}{4.9}$$

$$t \approx 10.1 s.$$

Not Required

To find velocity at impact, set $t \approx 10.1$ in (1).

$$V = 9.8t$$

$$V = 9.8(10.1)$$

$$V = 98.98 \text{ m/s}.$$

3-0235 — 50 SHEETS — 5 SQUARES
3-0236 — 100 SHEETS — 5 SQUARES
3-0237 — 200 SHEETS — 5 SQUARES
3-0137 — 200 SHEETS — FILLER

COMET

5pts

5 ~~If~~

$$f(x) = \int_0^{2x^2} t \sin t \, dt,$$

let $u = 2x^2$, then

$$f(x) = \int_0^u t \sin t \, dt.$$

Then, by the chain rule,

$$f'(x) = \frac{d}{dx} \int_0^u t \sin t \, dt$$

$$= \frac{d}{du} \int_0^u t \sin t \, dt \cdot \frac{du}{dx}$$

$$= (u \sin u) \cdot (4x)$$

Subbing back $u = 2x^2$,

$$f'(x) = (2x^2 \sin 2x^2)(4x)$$

$$= 8x^3 \sin 2x^2$$

3-0235 — 50 SHEETS — 5 SQUARES
3-0236 — 100 SHEETS — 5 SQUARES
3-0237 — 200 SHEETS — 5 SQUARES
3-0137 — 200 SHEETS — FILLER

COMET

5 pts

6a $\int_0^1 (x^2-2x)\,dx$

$= \frac{1}{3}x^3 - x^2 \Big|_0^1$

$= \left[\frac{1}{3}(1)^3-(1)^2\right] - \left[\frac{1}{3}(0)^3-(0)^2\right]$

$= \frac{1}{3} - 1$

$= -\frac{2}{3}$

5 pts

6b $\int_0^4 x\sqrt{x}\,dx$

$= \int_0^4 x \cdot x^{1/2}\,dx$

$= \int_0^4 x^{3/2}\,dx$

$= \frac{2}{5}x^{5/2}\Big|_0^4$

$= \frac{2}{5}\left[4^{5/2} - 0^{5/2}\right]$

$= \frac{2}{5}\left[32\right]$

$= \frac{64}{5} \approx 12.8$

5 pts

6c $\int_0^4 x\sqrt{9+x^2}\,dx$

Let $u = 9+x^2$.

$du = \frac{du}{dx}\,dx$

$du = 2x\,dx$

@ $x=0 \Rightarrow u=9$

@ $x=4 \Rightarrow u=25$

$= \frac{1}{2}\int_0^4 \sqrt{9+x^2}\,(2x\,dx)$

$= \frac{1}{2}\int_9^{25} \sqrt{u}\,du$

$= \frac{1}{2}\int_9^{25} u^{1/2}\,du$

$= \frac{1}{2}\left(\frac{2}{3}u^{3/2}\right)\Big|_9^{25}$

$= \frac{1}{3}\left[25^{3/2} - 9^{3/2}\right]$

$= \frac{1}{3}\left[125 - 27\right]$

$= \frac{98}{3}$

$= 32\frac{2}{3}$

~~5 pts~~

6d) $\int_0^1 \frac{x}{1+2x^2}\,dx$

Let $u = 1+2x^2$.

$du = \frac{du}{dx}\,dx$

$du = 4x\,dx$

$= \frac{1}{4}\int_0^1 \frac{1}{1+2x^2}(4x\,dx)$

@ $x=0 \Rightarrow u=1$

@ $x=1 \Rightarrow u=3$

$= \frac{1}{4}\int_1^3 \frac{1}{u}\,du$

$= \frac{1}{4}\ln|u|\Big|_1^3$

$= \frac{1}{4}\{\ln 3 - \ln 1\}$

$= \frac{1}{4}\ln 3$