

College of the Redwoods  
Mathematics Department

Differential Calculus — Math 50A  
Review for Final Exam

## Exam Questions

**Instructions.** Place the solution to each of the following questions on your own paper. You must show all of your work to receive credit for your solution. Staple the quiz to your solutions before submitting your quiz.

EXERCISE 1. Consider the polynomial function

$$p(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 6x + 8.$$

- Use the first derivative test to determine where the function is increasing and decreasing. Summarize your results on a number line as demonstrated in class. Below the line, note where the derivative is positive or negative, above the line note where the function is increasing or decreasing.
- Use the result of part (a) to determine the coordinates of any extrema. Plot these accurately on your coordinate system on your graph paper.
- Use the second derivative to determine the concavity of the polynomial function. Summarize your results on a number line. Below the line, note where the second derivative is positive or negative, above the line note where the function is concave up or concave down.
- Use the result in part (c) to determine the coordinates of the point(s) of inflection. Plot these accurately on your coordinate system.
- Sketch the graph of the polynomial function using the information from parts (a)-(d).

EXERCISE 2. Use l'Hopital's Rule to evaluate each of the following limits.

(a)  $\lim_{x \rightarrow 0} \frac{\sin 2x}{3x}$

(b)  $\lim_{x \rightarrow 0} (1+x)^{2/x}$

EXERCISE 3. The volume of a closed cylindrical can is  $125 \text{ in}^3$ . Find the radius  $r$  and the height  $h$  of the can having minimal surface area (the top, bottom, and lateral area combined). *Note: Exact answers only. No decimals. Don't be surprised by messy radicals, etc., in your solution.*

EXERCISE 4. A ball is thrown upward from the top of a building that is 100 meters above ground level with an initial velocity of 200 meters per second. The acceleration due to gravity is 9.8 meters per second per second.

- At what time will the ball reach its maximum height?
- What will be the maximum height of the ball?
- At what time will the ball return to ground level?

EXERCISE 5. Find the first derivative of the function defined by

$$f(x) = \int_x^{x^2} \ln t \, dt.$$

EXERCISE 6. Evaluate each of the following indefinite integrals.

(a)  $\int (2x+1)^6 \, dx$

(b)  $\int \cos^3 x \sin x \, dx$

(c)  $\int x\sqrt{x+2} \, dx$

(d)  $\int x \csc^2 x^2 \, dx$

EXERCISE 7. Evaluate each of the following definite integrals.

(a)  $\int_0^{\pi/2} \sec^2 \frac{x}{2} dx$

(b)  $\int_0^{\pi/6} \sec 2x \tan 2x dx$

(c)  $\int_0^1 x2^{-x^2} dx$

(d)  $\int_0^{\pi/6} \frac{1}{1+4x^2} dx$