(1a) 5pts $\int_0^1 x^2 e^{-x}\,dx$

	D	I
$+$	x^2	e^{-x}
$-$	$2x$	$-e^{-x}$
$+$	2	e^{-x}
$-$	0	$-e^{-x}$

$$\int_0^1 x^2 e^{-x}\,dx = -x^2 e^{-x} - 2xe^{-x} - 2e^{-x}\Big|_0^1$$

$$= -e^{-x}(x^2+2x+2)\Big|_0^1$$

$$= -\left\{ e^{-1}(1+2+2) - e^0(0+0+2) \right\}$$

$$= -\left\{ 5e^{-1} - 2 \right\}$$

$$= -\frac{5}{e} + 2$$

$$= 2 - \frac{5}{e}$$

(14) $\int_0^{\pi/4} e^{-2\theta} \sin 2\theta \, d\theta$

First, let's integrate by parts to find an antiderivative.

	D	F
+	$e^{-2\theta}$	$\sin 2\theta$
−	$-2e^{-2\theta}$	$-\frac{1}{2}\cos 2\theta$
+	$4e^{-2\theta}$	$-\frac{1}{4}\sin 2\theta$

Hence,

$$\int e^{-2\theta} \sin 2\theta \, d\theta = -\frac{1}{2} e^{-2\theta} \cos 2\theta - \frac{1}{2} e^{-2\theta} \sin 2\theta - \int e^{-2\theta} \sin 2\theta \, d\theta$$

$$2 \int e^{-2\theta} \sin 2\theta \, d\theta = -\frac{1}{2} e^{-2\theta} (\cos 2\theta + \sin 2\theta)$$

$$\int e^{-2\theta} \sin 2\theta \, d\theta = -\frac{1}{4} e^{-2\theta} (\cos 2\theta + \sin 2\theta)$$

Hence,

$$\int_0^{\pi/4} e^{-2\theta} \sin 2\theta = -\frac{1}{4} e^{-2\theta} (\cos 2\theta + \sin 2\theta) \Big|_0^{\pi/4}$$

$$= \left[-\frac{1}{4} e^{-2(\pi/4)} \left(\cos 2\left(\tfrac{\pi}{4}\right) + \sin 2\left(\tfrac{\pi}{4}\right)\right)\right]$$

$$- \left[-\frac{1}{4} e^{-2(0)} \left(\cos 2(0) + \sin 2(0)\right)\right]$$

$$= -\frac{1}{4} e^{-\pi/2}\left(\cos \tfrac{\pi}{2} + \sin \tfrac{\pi}{2}\right)$$

$$+ \frac{1}{4} e^{0}\left(\cos 0 + \sin 0\right)$$

$$= -\frac{1}{4} e^{-\pi/2}(0+1) + \frac{1}{4}(1+0)$$

$$= -\frac{1}{4} e^{-\pi/2} + \frac{1}{4}$$

$$= \frac{1}{4}\left(1 - e^{-\pi/2}\right)$$

6 pts

2a $\int \sin^2 \frac{\theta}{2}\, d\theta$

$$= \int \frac{1-\cos\theta}{2}\, d\theta$$

$$= \frac{1}{2}\int (1-\cos\theta)\, d\theta$$

$$= \frac{1}{2}\left[\theta - \sin\theta\right] + C$$

$$= \frac{1}{2}\theta - \frac{1}{2}\sin\theta + C$$

5.3
(26)

$$\int \sin 2x \cos 7x \, dx$$

$$= \int \frac{1}{2} [\sin(2x+7x) + \sin(2x-7x)] \, dx$$

$$= \frac{1}{2} \int [\sin 9x + \sin(-5x)] \, dx$$

$$= \frac{1}{2} \int [\sin 9x - \sin 5x] \, dx$$

$$= \frac{1}{2} \left[-\frac{1}{9} \cos 9x + \frac{1}{5} \cos 5x \right] + C$$

$$= \frac{1}{10} \cos 5x - \frac{1}{18} \cos 9x + C.$$

5.5 ③

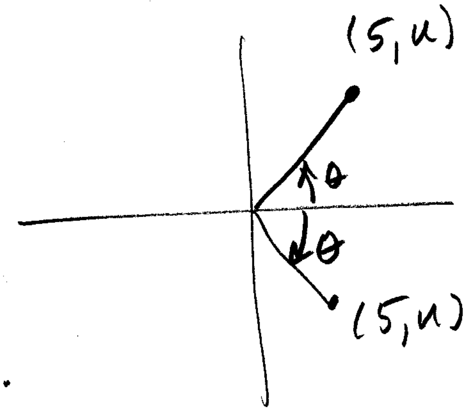
$$\int \frac{dx}{x^2+2x+26}$$

$$= \int \frac{dx}{x^2+2x+1-1+26}$$

$$= \int \frac{dx}{(x+1)^2+25}$$

$u = x+1$

$du = dx$



$$= \int \frac{du}{u^2+25}$$

$u = 5\tan\theta$

$du = 5\sec^2\theta\, d\theta$

$-\pi/2 < \theta < \pi/2$

$$= \int \frac{5\sec^2\theta\, d\theta}{25\tan^2\theta + 25}$$

$\tan\theta = \frac{u}{5}$

$\theta = \tan^{-1}\frac{u}{5}$

$$= \frac{5}{25}\int \frac{\sec^2\theta\, d\theta}{\tan^2\theta+1}$$

$$= \frac{1}{5}\int \frac{\sec^2\theta\, d\theta}{\sec^2\theta}$$

$$= \frac{1}{5}\int d\theta$$

$$= \frac{1}{5}\theta + C$$

$$= \frac{1}{5}\tan^{-1}\frac{u}{5} + C$$

$$= \frac{1}{5}\tan^{-1}\frac{x+1}{5} + C$$

5 pts

(4) $$\int \frac{dx}{(x-5)^2(x^2+4)}$$

Partial Fraction Decomposition

$$\frac{1}{(x-5)^2(x^2+4)} = \frac{A}{x-5} + \frac{B}{(x-5)^2} + \frac{Cx+D}{x^2+4}$$

$$1 = A(x-5)(x^2+4) + B(x^2+4) + (Cx+D)(x-5)^2$$

Let $x=5$. Then $1 = 29B$, so $B = \frac{1}{29}$

Expand and collect like terms:

$$1 = A(x^3-5x^2+4x-20) + Bx^2+4B + (Cx+D)(x^2-10x+25)$$

$$1 = \cancel{Ax^3} - \cancel{5Ax^2} + \cancel{4Ax} - \cancel{20A} + \cancel{Bx^2} + \cancel{4B} + \cancel{Cx^3} - \cancel{10Cx^2} + \cancel{25Cx}$$
$$+ \cancel{Dx^2} - \cancel{10Dx} + \cancel{25D}$$

Collecting:

$$1 = (A+C)x^3 + (-5A+B-10C+D)x^2$$
$$+ (4A+25C-10D)x + (-20A+4B+25D)$$

Hence,

$$A + C = 0$$

$$-5A + B - 10C + D = 0$$

$$4A + 25C - 10D = 0$$

$$-20A + 4B + 25D = 1$$

Set up augmented matrix:

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ -5 & 1 & -10 & 1 & 0 \\ 4 & 0 & 25 & -10 & 0 \\ -20 & 4 & 0 & 25 & 1 \end{bmatrix}$$

Reduce using Matlab or Calculator

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -10/841 \\ 0 & 1 & 0 & 0 & 1/29 \\ 0 & 0 & 1 & 0 & 10/841 \\ 0 & 0 & 0 & 1 & 21/841 \end{bmatrix}$$

$A = -10/841$

$B = 1/29$

$C = 10/841$

$D = 21/841$

Hence,

$$\int \frac{dx}{(x-5)^2(x^2+4)}$$

$$= \int \left[\frac{-10/841}{x-5} + \frac{1/29}{(x-5)^2} + \frac{(10/841)x + (121/841)}{x^2+4} \right] dx$$

$$= -\frac{10}{841} \int \frac{dx}{x-5} + \frac{1}{29} \int \frac{dx}{(x-5)^2} + \frac{1}{841} \int \frac{10x+21}{x^2+4}\, dx$$

$$= I_1 + I_2 + \frac{1}{841} \left[\int \frac{10x\,dx}{x^2+4} + \int \frac{21\,dx}{x^2+4} \right]$$

$$= I_1 + I_2 + \frac{1}{841} \left[5 \int \frac{2x\,dx}{x^2+4} + 21 \int \frac{dx}{x^2+4} \right]$$

$$= I_1 + I_2 + \frac{5}{841} \int \frac{2x\,dx}{x^2+4} + \frac{21}{841} \int \frac{dx}{x^2+4}$$

$$= I_1 + I_2 + I_3 + I_4$$

We'll now calculate I_1, I_2, I_3, and I_4 independently.

$$I_1 = -\frac{10}{841}\int \frac{dx}{x-5} = -\frac{10}{841}\ln|x-5|$$

$$I_2 = \frac{1}{29}\int \frac{dx}{(x-2)^2} = \frac{1}{29}\int (x-2)^{-2}\,dx$$

$$I_2 = \frac{1}{29}\left[-(x-2)^{-1}\right] = -\frac{1}{29}\cdot\frac{1}{x-2}$$

$$I_3 = \frac{5}{841}\int \frac{2x\,dx}{x^2+4} = \frac{5}{841}\ln(x^2+4)$$

$$I_4 = \frac{21}{841}\int \frac{dx}{x^2+4}$$

$x = 2\tan\theta$, $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$dx = 2\sec^2\theta\,d\theta$

$$= \frac{21}{841}\int \frac{2\sec^2\theta\,d\theta}{4\tan^2\theta+4}$$

$$= \frac{21}{841}\cdot\frac{2}{4}\int \frac{\sec^2\theta\,d\theta}{\tan^2\theta+1}$$

$$I_4 = \frac{21}{841} \cdot \frac{1}{2} \int \frac{\sec^2\theta \, d\theta}{\sec^2\theta}$$

$$= \frac{21}{1682} \int d\theta$$

$$= \frac{21}{1682} \theta$$

$$= \frac{21}{1682} \tan^{-1} \frac{x}{2}$$

But $x = 2\tan\theta$,

$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$,

so $\theta = \tan^{-1} \frac{x}{2}$

Hence, $\displaystyle\int \frac{dx}{(x-5)^2(x^2+4)}$

$$= I_1 + I_2 + I_3 + I_4$$

$$= -\frac{10}{841} \ln|x-5| - \frac{1}{29(x-2)}$$

$$+ \frac{5}{841} \ln(x^2+4) + \frac{21}{1682} \tan^{-1}\frac{x}{2} + C$$