College of the Redwoods
Mathematics Department

Intermediate Algebra— Math 120
Review Notes Exam #4

Last Revision Date: October 22, 2007 Version 1.00

Multiple Choice Questions

Instructions: *For each of the following questions, select the "best" answer and darken the corresponding oval on your scantron. Good luck!*

1. If

$$f(x) = \begin{cases} -3, & \text{if } x < 0 \\ 2x - 3, & \text{if } x \geq 0, \end{cases}$$

evaluate $f(-3)$.

(a) -3 (b) 0 (c) -9

(d) 1 (e) None of these

2. If f is the piecewise function of Exercise 1, evaluate $f(2)$.

(a) -3 (b) 0 (c) -9

(d) 1 (e) None of these

3. If $x < 1$, then $|x - 1|$ equals

(a) $x + 1$ (b) $x - 1$ (c) $-x + 1$

(d) $-x - 1$ (e) None of these

4. If $x < 0$, then $f(x) = x - |x|$ equals

(a) $2x$ (b) $-2x$ (c) 0

(d) x (e) None of these

5. Which of the following best describes the solution of $|x| < 7$?

(a) $(-7, 7)$ (b) $(-\infty, -7) \cup (7, \infty)$ (c) $[-7, 7]$

(d) $(-\infty, -7] \cup [7, \infty)$ (e) None of these

6. Which of the following best describes the solution of $|x| \geq 7$?

(a) $(-7, 7)$ (b) $(-\infty, -7) \cup (7, \infty)$ (c) $[-7, 7]$

(d) $(-\infty, -7] \cup [7, \infty)$ (e) None of these

7. Which of the following best describes the solution of $|x - 3| < -9$?

(a) $(-\infty, \infty)$ (b) No solutions (c) $(-6, 6)$

(d) $(-\infty, -6) \cup (6, \infty)$

8. What is the equation of the axis of symmetry of $y = (x - 2)^2 + 3$?

(a) $y = 3$ (b) $y = 2$ (c) $x = -2$

(d) $x = 2$ (e) None of these

9. What are the coordinates of the vertex of the parabola $y = 2x^2 - x - 1$?

(a) $(1/4, 9/8)$ (b) $(-1/4, 9/8)$ (c) $(1/4, -9/8)$

(d) $(-1/4, -9/8)$ (e) None of these

10. One of the solutions of $x^2 - 2x - 4 = 0$ is

(a) $(2 + \sqrt{2})/2$ (b) $(2 + \sqrt{12})/2$ (c) $(2 + \sqrt{20})4$

(d) $(2 + \sqrt{20})/2$ (e) None of these

11. Find k so that the parabola $y = kx^2 - 5x - 6$ has exactly one x-intercept.

(a) 24/25 (b) 36/25 (c) 25/24

(d) -1 (e) None of these

12. Find the maximum value of the function $f(x) = -x^2 + 2x + 6$.

(a) 6 (b) 7 (c) 8

(d) 9 (e) None of these

13. At what value of x does the function $f(x) = x^2 - 3x - 9$ achieve its maximum value?

(a) $-11/25$ (b) -9 (c) -1.5

(d) 1.5 (e) None of these

Instructions. *Please place the solution of each of the following questions on your own paper. You must show all supporting work to receive credit for your solution. If you are asked to write a response, grammar, spelling, punctuation, and style are important.*

Exercise 1. Sketch each of the given piecewise functions.

(a) $f(x) = \begin{cases} -2, & \text{if } x < 0 \\ 2, & \text{if } x \geq 0 \end{cases}$ (b) $f(x) = \begin{cases} -1, & \text{if } x < 0 \\ 2 - x, & \text{if } x \geq 0 \end{cases}$

Exercise 2. For each of the following functions, perform each of the following tasks.

- Create a number line. Place the expression inside the absoute value bars below and to the left of the number line. Place the function itself above and to the left of the number line. Mark the critical value on the number line, then determine the sign of the expression inside the absolute value bars on each side of the number line. Above the number line, simplify the function by removing the absolute value bars on each side of the critical value.
- Use your number line summary to help write a piecewise definition for the given function.
- Sketch the graph of the piecewise function on graph paper.

(a) $f(x) = |x + 5|$ (b) $f(x) = |5 - 2x|$

Exercise 3. For each of the following inequalities, perform each of the following tasks.

- Load each side of the inequality into your graphing calculator and sketch the graph. Copy the result onto your examination paper and label each graph with its equation. Use the `intersect` utility on your calculator to determine the points of intersection. Shade and label the solution of the inequality on the x-axis of your image.
- Solve the given inequality algebraically. Shade and label your solution on a separate number line, then use interval notation to describe your solution.

(a) $|2x - 5| > 11$ (b) $\left| x - \dfrac{x+5}{3} \right| < 1$

Exercise 4. Solve each of the following inequalities algebraically.

(a) $3 + 2|x - 5| > 5|x - 5| + 1$ (b) $5 - 4|x - 3| \leq 1$

Exercise 5. For each of the following quadratic functions, perform each of the following tasks.

- Complete the square to place the function in vertex form. On graph paper, plot the vertex and label it with its coordinates. Draw the axis of symmetry and label it with its equation.
- Plot the y-intercept on your plot and label it with its coordinates.
- Use the ac-test to factor the given quadratic function. Set the result equal to zero and solve for x. Plot the x-intercepts of the parabola on your plot and label each with its coordinates.
- Draw the parabola. Use interval notation to describe the domain and range of the parabola.

(a) $f(x) = x^2 - 6x - 16$

(b) $f(x) = -2x^2 + 13x + 24$

Exercise 6. For each of the following quadratic functions, perform each of the following tasks.

- Complete the square to place the function in vertex form. On graph paper, plot the vertex and label it with its coordinates. Draw the axis of symmetry and label it with its equation.
- Plot the y-intercept on your plot and label it with its coordinates.
- Use the quadratic formula to find the zeros of the given function. Approximate each to the nearest tenth. Plot the x-intercepts of the function, then label them with their **exact** coordinates.
- Draw the parabola. Use interval notation to describe the domain and range of the parabola.

(a) $f(x) = x^2 - 6x - 10$

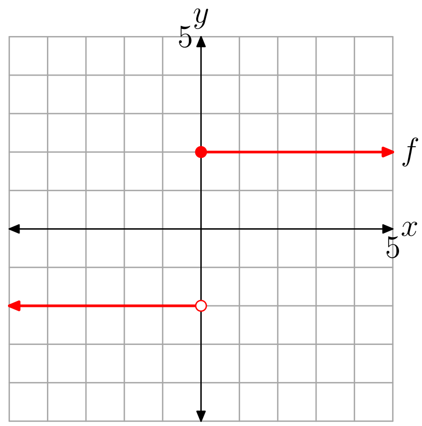
(b) $f(x) = -x^2 - 5x + 12$

Exercise 7. Find two numbers whose difference is 12 so that the sum of their squares is a minimum.

Exercise 8. One number is 5 less than twice a second number. Find two such numbers so that their product is a minimum.

Solutions to Exercises

Exercise 1(a)



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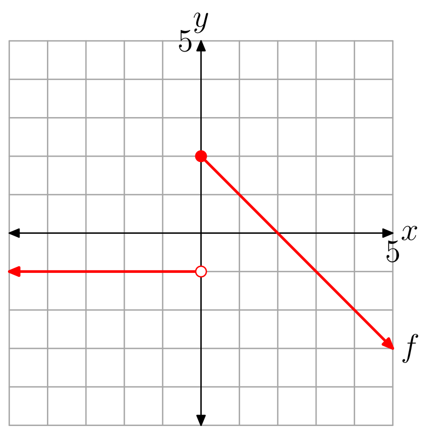
Exercise 1(b)



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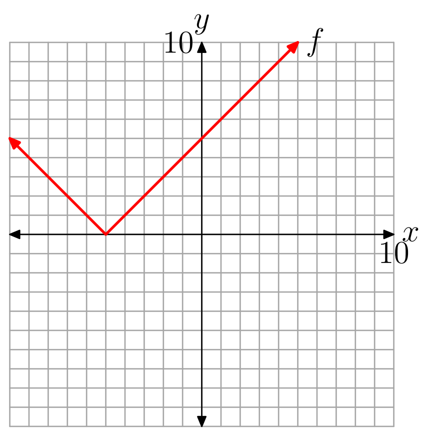
Exercise 2(a)

$$f(x) = \begin{cases} -x-5, & \text{if } x < -5 \\ x+5, & \text{if } x \geq -5 \end{cases}$$



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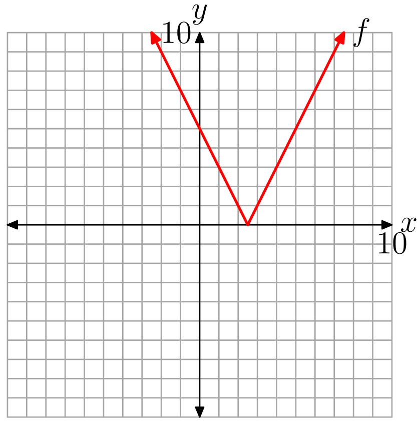
Exercise 2(b)

$$f(x) = \begin{cases} 5 - 2x, & \text{if } x < 5/2 \\ -5 + 2x, & \text{if } x \geq 5/2 \end{cases}$$



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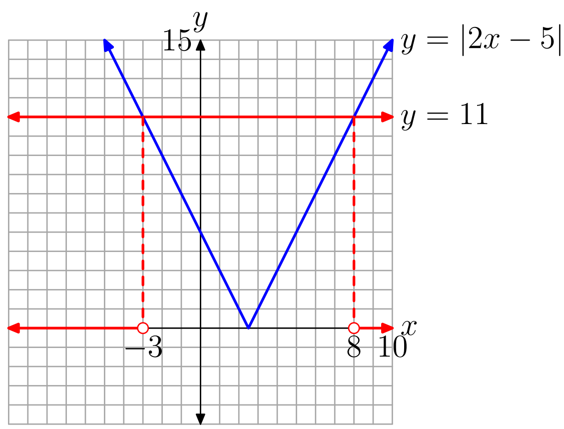
Exercise 3(a)



Write

$$|2x - 5| > 11$$

in the equivalent form

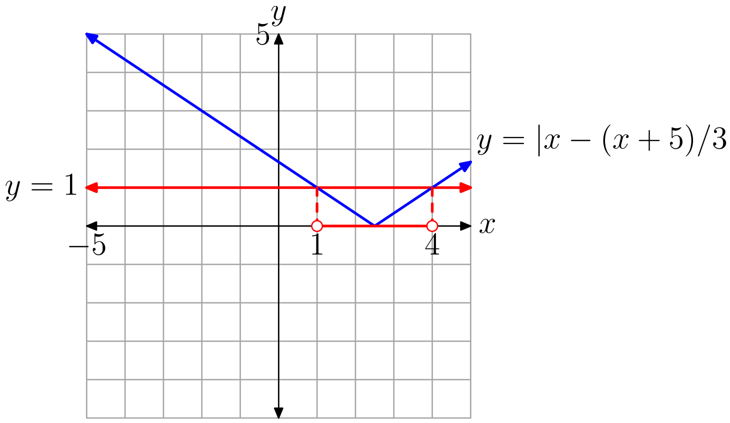
$$2x - 5 < -11 \quad \text{or} \quad 2x - 5 > 11$$

Solve each side independently. Add 5, then divide by 2.

$$\begin{aligned} 2x &< -6 \\ x &< -3 \end{aligned} \qquad \text{or} \qquad \begin{aligned} 2x &> 16 \\ x &> 8 \end{aligned}$$

In interval notation, the solution is $(-\infty, -3) \cup (8, \infty)$. □

Exercise 3(b)



Multipy each side

$$\left| x - \frac{x+5}{3} \right| < 1$$

by 3.

$$\begin{aligned} |3x - (x+5)| &< 3 \\ |3x - x - 5| &< 3 \\ |2x - 5| &< 3 \end{aligned}$$

Write this last result in the equivalent form

$$-3 < 2x - 5 < 3. First add 5 to all three sides, then divide all three sides by 2.$$

Solve.

$$\begin{aligned} 2 < 2x < 8 \\ 1 < x < 4 \end{aligned}$$

In interval notation, the solution is $(1, 4)$. □

Exercise 4(a) Isolate the absolute values on one side of the equation.

$$\begin{aligned} 3 + 2|x-5| &> 5|x-5| + 1 \\ 2|x-5| - 5|x-5| &> 1 - 3 \\ -3|x-5| &> -2 \end{aligned}$$

Divide by -3, reversing the inequality.

$$|x - 5| < \frac{2}{3}$$

Write in equivalent form and solve.

$$\begin{aligned} -\frac{2}{3} < x - 5 < \frac{2}{3} \\ -\frac{2}{3} < x - \frac{15}{3} < \frac{2}{3} \\ \frac{13}{3} < x < \frac{17}{3} \end{aligned}$$

□

Exercise 4(b) Isolate the absolute value.

$$\begin{aligned} 5 - 4|x-3| &\le 1 \\ -4|x-3| &\le 1 - 5 \\ -4|x-3| &\le -4 \end{aligned}$$

Divide both sides by -4, reversing the inequality.

$$|x-3| \ge 1$$

Write as an equivalent inequality and solve.

$$\begin{aligned} x - 3 &\le -1 \quad &\text{or}\quad && x - 3 &\ge 1 \\ x &\le 2 &&& x &\ge 4 \end{aligned}$$

□

Exercise 5(a) To place $f(x) = x^2 - 6x - 16$ in vertex form, take one-half of the middle coefficient and square, i.e., $[(1/2)(-6)]^2 = 9$. Add and subtract this amount, factor and simplify.

$$\begin{aligned} f(x) &= x^2 - 6x + 9 - 9 - 16 \\ f(x) &= (x-3)^2 - 25 \end{aligned}$$

The parabola opens upward, the vertex is at $(3, -25)$, and the equation of the axis of symmetry is $x = 3$.

Because $f(0) = -16$, the y-intercept is $(0, -16)$. To find the x-intercept, set $y = 0$.

$$0 = x^2 - 6x - 16$$

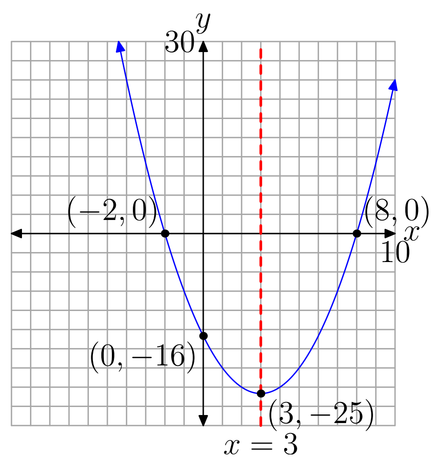
Note that $ac = (1)(-16) = -16$. The integer pair $\{2, -8\}$ has product -16 and sums to -6. Hence,

$$0 = (x+2)(x-8).$$

Set each factor equal to zero and solve.

$$\begin{aligned} x + 2 &= 0 \quad &\text{or}\quad && x - 8 &= 0 \\ x &= -2 &&& x &= 8 \end{aligned}$$

Thus, the x-intercepts are $(-2, 0)$ and $(8, 0)$.



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Exercise 5(b) To place $f(x) = -2x^2 + 13x + 24$ in vertex form, first factor out a -2.

$$f(x) = -2\left[x^2 - \frac{13}{2}x - 12\right]$$

Next, take one-half of the middle coefficient and square, i.e., $[(1/2)(-13/2)] = 169/16$. Add and subtract this amount, factor and simplify.

$$\begin{aligned}
f(x) &= -2\left[x^2 - \frac{13}{2}x + \frac{169}{16} - \frac{169}{16} - 12\right] \\
f(x) &= -2\left[\left(x - \frac{13}{4}\right)^2 - \frac{169}{16} - \frac{192}{16}\right] \\
f(x) &= -2\left[\left(x - \frac{13}{4}\right)^2 - \frac{361}{16}\right] \\
f(x) &= -2\left(x - \frac{13}{4}\right)^2 + \frac{361}{8}
\end{aligned}$$

The parabola opens downward, the vertex is at $(13/4, 361/8)$, and the equation of the axis of symmetry is $x = 13/4$.

Because $f(0) = 24$, the y-intercept is $(0, 24)$. To find the x-intercept, set $y = 0$, then multiply both sides by -1.

$$\begin{aligned}
0 &= -2x^2 + 13x + 24 \\
0 &= 2x^2 - 13x - 24
\end{aligned}$$

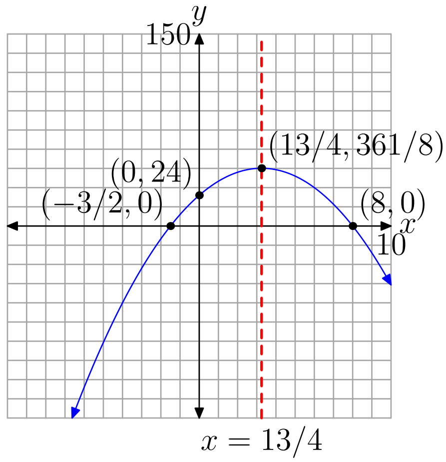
Note that $ac = (2)(-24) = -48$. The integer pair $\{3, -16\}$ has product -48 and sums to -13. Hence,

$$\begin{aligned}
0 &= 2x^2 + 3x - 16x - 24 \\
0 &= x(2x + 3) - 8(2x + 3) \\
0 &= (x - 8)(2x + 3).
\end{aligned}$$

Set each factor equal to zero and solve.

$$\begin{aligned}
x - 8 &= 0 \qquad\qquad \text{or} \qquad\qquad & 2x + 3 &= 0 \\
x &= 8 & x &= -3/2
\end{aligned}$$

Thus, the x-intercepts are $(-3/2, 0)$ and $(8, 0)$.



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Exercise 6(a) To place $f(x) = x^2 - 6x - 10$ in vertex form, take one-half of the middle coefficient and square, i.e., $[(1/2)(-6)] = 9$. Add and subtract this amount, factor and simplify.

$$f(x) = x^2 - 6x + 9 - 9 - 10$$
$$f(x) = (x-3)^2 - 19$$

The parabola opens upward, the vertex is at $(3, -19)$, and the equation of the axis of symmetry is $x = 3$.

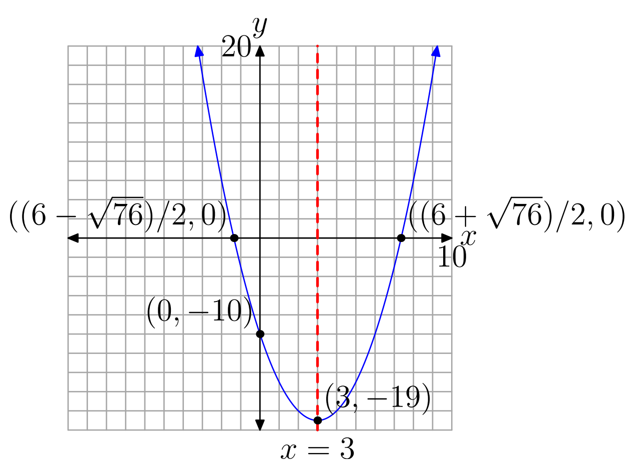
Because $f(0) = -10$, the y-intercept is $(0, -10)$. To find the x-intercept, set $y = 0$.

$$0 = x^2 - 6x - 10$$

Note that $ac = (1)(-10) = -10$. There is no integer pair that has product -10 and sums to -6. Hence, we will need to use the quadratic formula.

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-10)}}{2(1)} \\ &= \frac{6 \pm \sqrt{76}}{2} \end{aligned}$$

Thus, the x-intercepts are $((6-\sqrt{76})/2)$ and $((6+\sqrt{76})/2)$. Using a calculator, these approximately equal -1.35 and 7.35.



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Exercise 6(b) To place $f(x) = -x^2 - 5x + 12$ in vertex form, first factor out a -1.

$$f(x) = -\left[x^2 + 5x - 12\right]$$

Take one-half of the middle coefficient and square, i.e., $[(1/2)(5)] = 25/4$. Add and subtract this amount, factor and simplify.

$$\begin{aligned} f(x) &= -\left[x^2 + 5x + \frac{25}{4} - \frac{25}{4} - \frac{48}{4}\right] \\ f(x) &= -\left[\left(x + \frac{5}{2}\right)^2 - \frac{73}{4}\right] \\ f(x) &= -\left(x + \frac{5}{2}\right)^2 + \frac{73}{4} \end{aligned}$$

The parabola opens downward, the vertex is at $(-5/2, 73/4)$, and the equation of the axis of symmetry is $x = -5/2$.

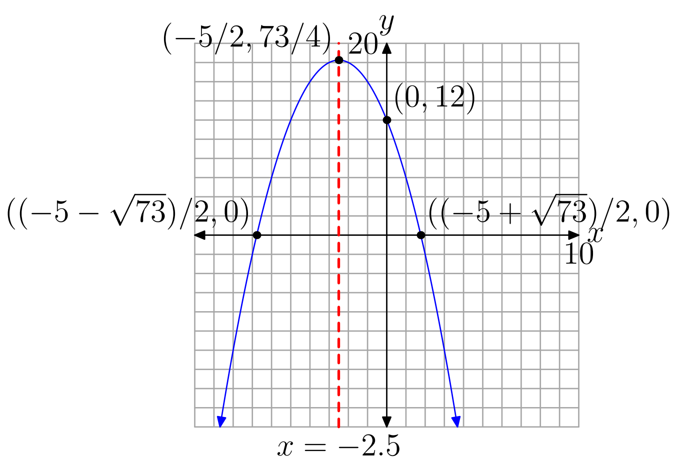
Because $f(0) = 12$, the y-intercept is $(0, 12)$. To find the x-intercept, set $y = 0$, then multiply both sides of the equation by -1.

$$\begin{aligned} 0 &= -x^2 - 5x + 12 \\ 0 &= x^2 + 5x - 12 \end{aligned}$$

Note that $ac = (1)(-12) = -12$. There is no integer pair that has product -12 and sums to 5. Hence, we will need to use the quadratic formula.

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-5 \pm \sqrt{(5)^2 - 4(1)(-12)}}{2(1)} \\ &= \frac{-5 \pm \sqrt{73}}{2} \end{aligned}$$

Thus, the x-intercepts are $((-5 - \sqrt{73})/2)$ and $((-5 + \sqrt{73})/2)$. Using a calculator, these approximately equal -6.7720 and 1.7720.



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Exercise 7. Let x and y represent numbers. Their difference is 12, so

$$x - y = 12. \tag{1}$$

The sum of their squares is

$$S = x^2 + y^2. \tag{2}$$

Solve equation (1) for x.

$$x = y + 12 \tag{3}$$

Substitute equation (3) into equation (2), expand and simplify.

$$\begin{aligned} S &= (y + 12)^2 + y^2 \\ S &= y^2 + 24y + 144 + y^2 \\ S &= 2y^2 + 24y + 144 \end{aligned}$$

This is a parabola that opens upward, so the minimum S-value will occur at the vertex. The y-value of the vertex is given by

$$y_v = -\frac{b}{2a} = -\frac{24}{2(2)} = -6.$$

To find the second number, substitute $y = -6$ in equation (3).

$$\begin{aligned} x &= -6 + 12 \\ x &= 6 \end{aligned}$$

Thus, the numbers are $x = 6$ and $y = -6$. Exercise 7

Exercise 8. One number is 5 less than twice a second number. So, the numbers are $2x - 5$ and x. The product of the two numbers is given by

$$\begin{aligned} P &= (2x - 5)x \\ P &= 2x^2 - 5x. \end{aligned}$$

This is a parabola that opens upward, so P will have a minimum at the vertex of the parabola. The x-value of the vertex is given by

$$x = -\frac{b}{2a} = -\frac{-5}{2(2)} = \frac{5}{4}.$$

The first number is

$$\begin{aligned} 2x - 5 &= 2\left(\frac{5}{4}\right) - 5 \\ &= \frac{5}{2} - \frac{10}{2} \\ &= -\frac{5}{2}. \end{aligned}$$

Hence, the two numbers are $5/4$ and $-5/2$. Exercise 8

Solutions to Multiple Choice Questions

Solution to Question 1: -3

$\square$

Solution to Question 2: 1

$\square$

Solution to Question 3: $-x+1$

$\square$

Solution to Question 4: $2x$

$\square$

Solution to Question 5: $(-7, 7)$

$\square$

Solution to Question 6: $(-\infty, -7] \cup [7, \infty)$

$\square$

Solution to Question 7: No solutions

$\square$

Solution to Question 8: $x = 2$

$\square$

Solution to Question 9: $(1/4, -9/8)$

$\square$

Solution to Question 10: $(2 + \sqrt{20})/2$

$\square$

Solution to Question 11: None of these. The correct solution is $k = -25/24$.

$\square$

Solution to Question 12: 7

$\square$

Solution to Question 13: 1.5

$\square$