An Economic Application of Linear Algebra

The Leontief Open Production Model or Input-Output Analysis

Iris Jensen

Introduction

- Ever wonder how the government can predict a deeper recession when the airline industry shrinks or why certain antiquated industries are subsidized by the government?
- Input-Output analysis the modern name for the process of manipulating a Leontief open production model.
- Input-output analysis can be applied to any size economy from a business district to the entire world.
- It is most often used for city planning and analysis of our national economy.
- The Leontief Open Production Model provides us with a powerful economic analysis tool.

Assigning Variables

- The technology matrix A will describe the relations a sector has with all the other sectors.

- The demand vector will be represented by D .

- The production vector X represents the total production that will be needed to satisfy the demand D.

A Simple Example

- This is a simple example of a technology matrix, The matrix is a 2×2.

$$A = \begin{bmatrix} .5 & .25 \\ .1 & .25 \end{bmatrix}$$

- a_{11} is the number of units in sector 1 that must be produced to meet a 1 unit increase in demand in sector 1.
- a_{12} is the number of units in sector 1 that must be produced to meet a 1 unit increase in demand in sector 2.
- a_{21} is the number of units in sector 2 that must be produced to meet a 1 unit increase in demand in sector 1.
- a_{22} is the number of units in sector 2 that must be produced to meet a 1 unit increase in demand in sector 2.

A Little Linear

- The production vector $X = (I - A)^{-1}D$. Where D is the external demand vector. Our example demand vector will be

$$D = \begin{bmatrix} 7000 \\ 14000 \end{bmatrix}.$$

- Since industries use products as they produce industries must produce more product than that demanded by the consumers.

$$X = \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} .5 & .25 \\ .1 & .25 \end{bmatrix} \right)^{-1} \begin{bmatrix} 7000 \\ 14000 \end{bmatrix} = \begin{bmatrix} 22000 \\ 24000 \end{bmatrix}$$

Goal

We will use the coefficient matrix from 1992 to analyze the interdependencies among the sectors. The most useful application of input-output analysis for the economist or a common broker is the ability to be able to see the value of a changes in demand for one or two industries on the entire economy. I will focus on showing that the same total value of increase of demand will have different effects on the Gross Domestic Product.

Code for GDP Program

```
for m=1:2
    Total_Old_Demand=Total_Old_Demand+Old_Economy_Demand(m,1);
end
Total_Old_Demand
for m=1:2
    Total_New_Demand=Total_New_Demand+New_Economy_Demand(m,1);
end
Total_New_Demand
Old_Production=inv(eye(2)-A)*Old_Economy_Demand; Old_Production
for m=1:2
    Total_Old_Production=Total_Old_Production+Old_Production(m,1);
end
Total_Old_Production
New_Production=inv(eye(2)-A)*D;
for m=1:2
    Total_New_Production=Total_New_Production+New_Production(m,1);
end
Total_New_Production
New_Production
for m=1:2
    Added_Production(m,1)=New_Production(m,1)-Old_Production(m,1);
end
```

```
Added_Production
for m=1:2
    Total_Growth=Total_Growth+ Added_Production(m,1);
end
Total_Growth
Percent_Change_in_GDP=(Total_Growth/Total_Old_Production)*100
```

Equally Weighted Demand Vectors Effects GDP Differently.

- the technology matrix we will use is for the 1992 U.S. economy and is 97×97, each column represents a different industry.
- I used four different examples all starting with a GDP of 219775812.49 million.
- I added 1000 million to ten different sectors.
- I used four different methods to pick the ten sectors.

Iris Picks Ten

- Sectors chosen were, 1, 5 and 6, 7, 9 and 10, 22 and 23, 44 and 45, 68 a, 73b, 75,also 78.
- It added 1027085.67 million to the GDP
- It caused a 1.15 percent increase.

Most Value Added

- Sectors chosen were, 4, 9 and 10, 15, 26 a, 65 a, 68 a, 69a, 69b, 71 a and 71 b.
- It added 122709.53 million to the GDP
- It caused a .14 percent increase.

Least Value Added

- Sectors chosen were, 1, 14, 25, 28, 31, 39, 51, 59 a, 59 b and 68 b.
- It added 94712.11 million to the GDP
- It caused a .11 percent increase.

Computer Ten Randomly

- Sectors chosen were, 2, 25, 43, 47, 49, 54, 65 d, 69 a, 73 b and 79.
- It added 56166.84 million to the GDP
- It caused a .06 percent increase.

Class Participation

- For the four examples I just showed I used the 97×97 coefficient matrix for 1992 entries are either zeros or five character five decimal place numbers.
- Now we will see if the class can do better than me and the computer.
- I will take ten sectors from the class and go into matlab to see the added production and it's effect on the GDP.

Conclusion

Although My idea that increasing sectors with the least value added would increase the GDP more than the sectors with the most value added did not hold up. It is obvious that the change in GDP is dependent on which sectors you choose. Input-Output analysis is a valuable tool for economic projections into the future. This application is much more heavy on data analysis than linear algebra.

References

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[4] Strang, Gilbert. **Introduction to Linear Algebra.** Wellesley-Cambridge Press, 1998.