Applications of Linear Algebra

Iterative Re-location of an Earthquake Hypocenter

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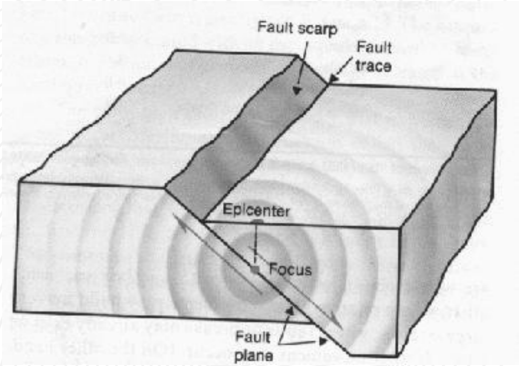
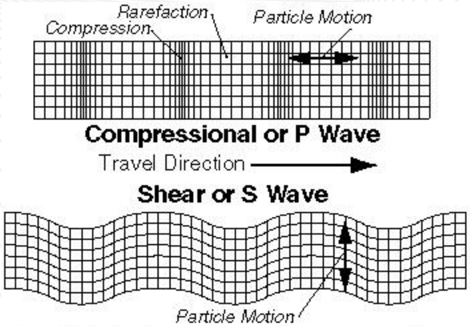
Introduction

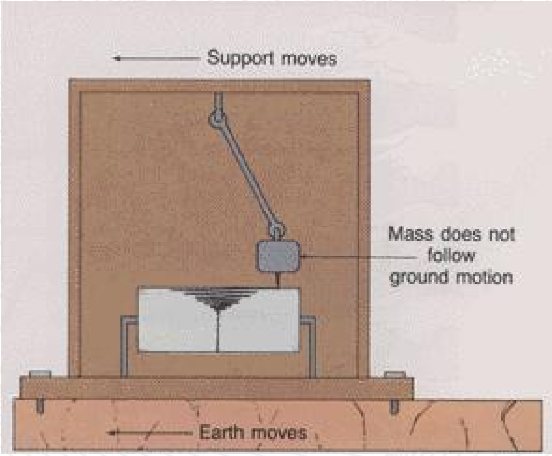
- An earthquake occurs...emitting multiple forms of energy



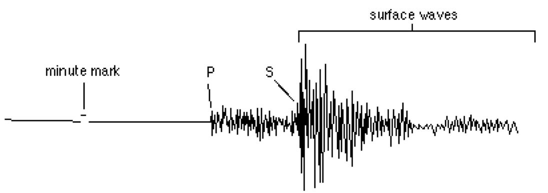


Seismograph

- method of observing earthquake:



- seismogram



P waves

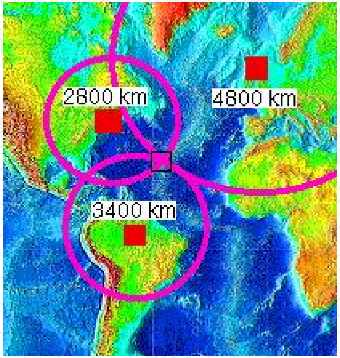
- P waves have the fastest arrival time
- The difference between P and S waves gives distance traveled
- Observed data: P wave arrival time
- this observed data builds our data vector, d

Goal

- use data to infer true (hypocenter) location and t_o

$$m_{true} = m_0 + \triangle m$$

- where m_0 is our original estimate of hypocenter and origin time (model vector)



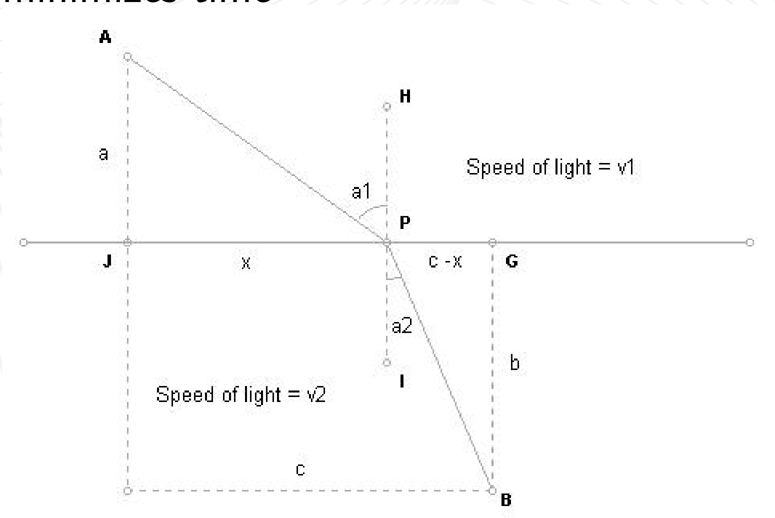
- Interpret subsurface.

Problem

- Non-linear; Travel time is a function of
  - path
  - velocity structure
  - original estimate of location
  - t_0
  - travel time equation for the i^{th} station
- Assumptions
  - P-wave velocity
  - hypocentral depth
  - wave ray path

Fermat's Principle of Least Time

- wave energy travels from hypocenter to station along the path which minimizes time



- taking the derivative of time wrt x (to find the minimum):

$$\frac{dt}{dx} = \frac{x}{v_1\sqrt{a^2 + x^2}} + \frac{c - x}{v_2\sqrt{b^2 + (c - x)^2}} v_2$$

Linearize Using Taylor's Series Expansion

- $T(m) \approx T(m_0) + \sum_{j=1}^{n} \frac{\partial d_i}{\partial m_j}(m - m_0) + \ldots$

- $\triangle T(m) \approx \sum_{j=1}^{n} \frac{\partial d_i}{\partial m_j} \triangle m$

Matrix equation:

(using n stations)

- $\triangle T(m) = G \triangle m$
  where T is $Nx1$ G is $Nx3$ and m is $3x1$

$$\begin{bmatrix} \triangle T_1 \\ \triangle T_2 \\ \vdots \\ \triangle T_n \end{bmatrix} = \begin{bmatrix} \partial & \partial & 1 \\ \partial & \partial & 1 \\ \vdots & \vdots & \vdots \\ \partial & \partial & 1 \end{bmatrix} \begin{bmatrix} \triangle x \\ \triangle y \\ \triangle t \end{bmatrix}$$

Solve for $\triangle m$ using SVD, Pseudo-inverse

- $G = U\Lambda V'$
- Since $G^{-1} = V\Lambda^{-1}U'$
- $G = U\Lambda V, G\dagger$

$$G^{-1} = V'\Lambda^{-1}U$$

$$therefore \triangle m = G^{-1} \triangle d$$

- Remember, $\triangle m$ adjusts m_0
- How do we know when our adjustment is sufficient?

What we have done:

- used arrival time data to find true location and origin time
  - latitude, longitude, and elevation of each station
  - P-wave arrival time
- knowing this information we can study regional plate tectonics
- interpret subsurface

more good stuff too!!!

Conclusion

THE END

References

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[2] Aki, Keiiti and Richards, Paul G. **Quantitative Seismology, Theory and Methods** W.H.Freeman and Company, 1980

[3] Menke, William C.**Geophysical Data Analysis: Discrete Inverse Theory** Academic Press, 1989

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[5] special thanks to:
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