Northern Spotted Owl Population Modeling Using the Leslie-Leftovich Matrix

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Abstract

We used the Leslie-Leftovich matrix to model the northern spotted owl population in the three different areas in the Pacific Northwest.

1 Introduction

In 1990, the northern spotted owl became the center of a nation wide controversy over the use and misuse of old growth forests from Northern California to Washington. In addition, the spotted owl habitat has been reduced by some 60% since 1800. As a result, only 2000 pairs of owls have been located in the Pacific Northwest during 1985-1990(in 1987, 2500 pairs of owls were identified). The federal government then in 1990 listed the spotted owl as an endangered species. The reduction of the owl habitat has in turn caused a severe decline in the owl population since owls are finding it harder and harder to find new areas to breed. Many areas in the Pacific Northwest, the owl population has been fragmented, a process that has isolated many populations of owls. This fragmentation , due to the clear-cutting of home ranges of the owl habitat, have exposed the owls to greater risks of predators while increasing the competition for scarce prey. Three areas that were used to model the northern spotted owl are: 1) the Klamath Province, in northwestern California; 2) The Olympic Peninsula in Washington; 3) and the Coast Range in western Oregon. The Leslie-Leftovich stage projection matrix was used, given the fecundity and survival rates of the owls, to determine the growth of the owl population and the age distribution with the population over time.

1.1 General Biology of the Northern Spotted Owl.

- Northern spotted Owls are monogamous breeders with low fecundity rates and high survival rates.
- They are territorial and tend to form long-term pair bonds. Breeding occurs irregularly.

- Owls inhabit the same breeding grounds for several year as long as nesting, roosting, and foraging habitat is present.
- Female owls rarely breed when only 1 year old; most do not begin until at least 2 years old.
- Young owls leave the nest in less than a year and need to find a mate and to establish their own territory.
- Nest sites are in old growth forests which are usually 200 feet high.
- Spotted owls live and average of 17 1/4 years; but can live up to 20 years.

1.2 Population dynamics of the spotted owl

- Life Cycle:

Juvenile	(up to 1 year)
Subadult	(1-2 years)
Adult	(over 2 years)

- Owls are widely distributed in forested regions of western Washington/Oregon and northwestern California in primarily mature and old conifer forests (>= 200 years old).
- Each owl requires about 4 square miles for its own home territory.
- Critical time in the owl life cycle occurs when the juveniles leave the nest to survive and become a subadult. During this critical time, a juvenile owl must successfully find a new home range and a mate.

1.3 Factors Affecting Population trends/dynamics

- Variation in nesting attempts and nesting success.
- Annual variation in percent of spotted owls paired.
- Percent of paired spotted owls that attempt breeding.
- Mortality in nest.
- Nestling mortality
- Rates of successful dispersal by juveniles

  (Variations in these rates and other factors can influence the likelihood of persistence of the population over time.)

Most Important for survival Dispersal of owls – relative permanent movement of owls from one location to another, usually juveniles from their nest area to a breeding site or occasionally adults from one breeding site to another. Without the successful replacement of owls that are lost from the breeding population during migration, the population will decline.

1.4 Mathematical Analysis Used in a Population Model Matrix

1.4.1 Definition and Illustration of an Eigenvalue and an Eigenvector

If $T : R^n \Longrightarrow R^n$ then $T(x) = Ax$ for some $N \times N$ matrix A. If $x \neq 0$ and $T(x) = A(x)$ is a scalar multiple of x that is if:

$$Ax = \lambda x$$

for some scalar λ, then λ is said to be an *eigenvalue* of T (or equivalently of A). Any nonzero vector x which satisfies this equation is said to be an *eigenvector* of T (or of A corresponding to λ). Suppose $T :\ R^2 \Rightarrow R^2$ defined by the equation:

$$T(x) = \begin{bmatrix} 1 & -2 \\ 3 & -4 \end{bmatrix} x$$

that is, T is given by the left multiplication by the matrix

$$A = \begin{bmatrix} 1 & -2 \\ 3 & -4 \end{bmatrix}$$

The image of the vector $x = (1,3)^T$ under the action of T:

$$T \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 3 & -4 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} -5 \\ 9 \end{bmatrix}$$

Clearly, $T(x)$ is not a scalar multiple of x, and this is what typically occurs.

Now consider the image of the vector $x = (2,3)^T$ under the action of T:

$$T \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 3 & -4 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} -4 \\ 6 \end{bmatrix}$$

Here $T(x)$ is a scalar multiple of x, since $T(x) = (-4,-6)^T;\ -2(2,3)^T = -2x$. Therefore -2 is an *eigenvalue* of T , and $(2,3)^T$ is an *eigenvector* corresponding to this *eigenvalue.*

1.5 Determining the Eigenvalues of a Matrix

Given a square matrix A, the condition that characterizes an eigenvalue λ, is the existence of a nonzero vector x such that $Ax = \lambda x$; this equation can be rewritten as follows:

$$\begin{aligned} Ax &= \lambda x \\ Ax - \lambda x &= 0 \\ Ax - \lambda I x &= 0 \\ (A - \lambda I)x &= 0 \end{aligned}$$

This final form of the equation makes it clear that x is the solution of a square, homogeneous system. If "nonzero" solutions are desired, then the *determinant* of the coefficient matrix which in this case is $A - \lambda I$ –must be **zero**; if not, then the system possesses only the trivial solution $x = 0$. Since eigenvectors are, by definition, nonzero, in order for x to be an eigenvector of a matrix A, λ must be chosen so that:

$$\det(A - \lambda I) = 0$$

When the determinant of $A - \lambda I$ is written out, the resulting expression is a monic polynomial in λ, [a monic polynomial is one in which the coefficient of the leading (the highest degree) term is 1]. It is called the *characteristic polynomial* of A– that is, the solution of a the *characteristic equation* $\det(A - \lambda I) = 0$ –are the *eigenvalues* of a matrix.

Determining the Eigenvectors of a Matrix In order to determine the *eigenvectors* of a matrix, you must determine the eigenvalues. Substitute one eigenvalue λ into the equation $Ax = \lambda x$ — or equivalently, into $(A - \lambda I)x = 0$ — and solve for x; the resulting **nonzero** solutions form the set of eigenvectors of A corresponding to the selected *eigenvalue*. This process is then repeated for each of the remaining eigenvalues.

1.6 Applying the Matrix Algebra to the Northern Spotted Owl Population

Using the population model of the spotted owl, the basic characteristic polynomial is given by:

$$\lambda^2 - s\lambda - s_0 s_1 b$$

from the basic matrix A :

$$\begin{bmatrix} 0 & \mathrm{s}_1\mathrm{b} & \mathrm{s}_p \\ \mathrm{s}_0 & 0 & 0 \\ 0 & \mathrm{s}_1 & \mathrm{s} \end{bmatrix}$$

The dominant (or greatest in absolute value), *eigenvalue*, λ_1 , is a real world solution of the estimate of the annual rate of change in the owl population. If $\lambda_1 > 1.0$, the vital rates suggest the population was increasing. If $\lambda_1 = 1.0$, the population size was stable; and if $\lambda_1 < 1.0$, the population was declining.

The *eigenvector* that corresponds to the dominant *eigenvalue* is used to further model the owl population by determining the relative proportion of female owls in each age class over a long period of time.

1.7 Leslie-Lefkovitch Matrix

The matrix we used for our population modeling is a hybrid of the Leslie matrix and the Lefkovitch matrix. The Leslie matrix works best for short lived species and where precise and specific estimates are available. The Leslie matrix stages classes according to age while the Lefkovitch divides the classes in to size or stages such as Juvenile, Subadult, and Adult. Often data is limited for long lived species, and estimates of age specific information are difficult to obtain. Repeated use of imprecise estimates can lead to large errors.

1.8 Matrix

Leslie-Lefkovitch matrix for the Northern Spotted Owl

$$\begin{bmatrix} J_{t+1} \\ S_{t+1} \\ A_{t+1} \end{bmatrix} = \begin{bmatrix} 0 & F_s & F_a \\ P_j & 0 & 0 \\ 0 & P_s & P_a \end{bmatrix} \qquad \begin{bmatrix} J_t \\ S_t \\ A_t \end{bmatrix}$$

J	Juvenile ($x < 1$ *year*)
S	Subadults($1 \leq x < 2$)
A	Adults($x \geq 2$)
F	Fecundity(reproduction rate)
P	Survival rate

We applied estimates for the fecundity and survival rates for three areas: Klamath in Northern California, Western Oregon , and the Olympic Peninsula in Washington

The Leslie-Lefkovitch model is given by the equation $x^k = L^k X^o$. The initial population distribution vector is $X^o$and the population distribution vector is x^k at time k. Our basic model has fecundity rates across the first row. The second row has the survival rates of the juvenile class and the third row contains the survival rates of the subadults and adults in column 2 & 3.

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1.9 Model Assumptions

- There is a constant reproduction rate for females over 2 years old.

- Birth population is an even split, male/ female.
- Stable age distribution exists.
- There is no density dependence. The change in population density doesn't effect fecundity rate.
- Constant adult survival and old age doesn't allow a decline in breeding.
- New population is generated at the same time yearly.

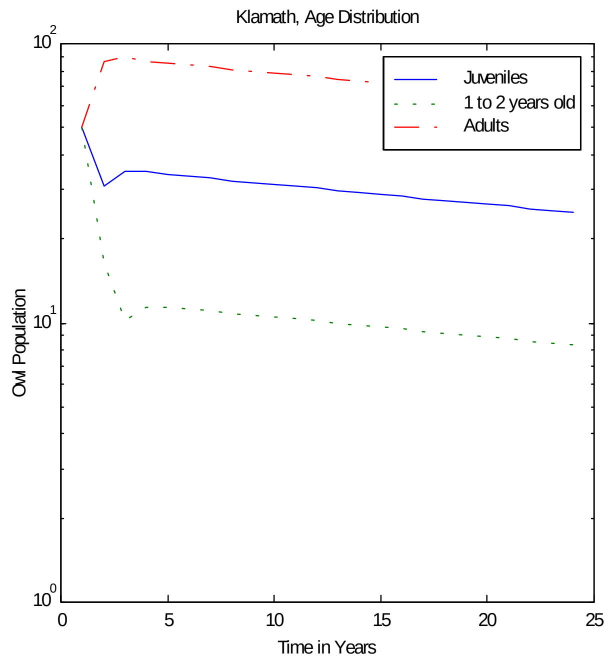
1.9.1 Klamath Province, Northwestern California

	Fecundity	Survivial
Juvenile	.09	.325
Subadults	.20	.8677
Adults	.33	.8667

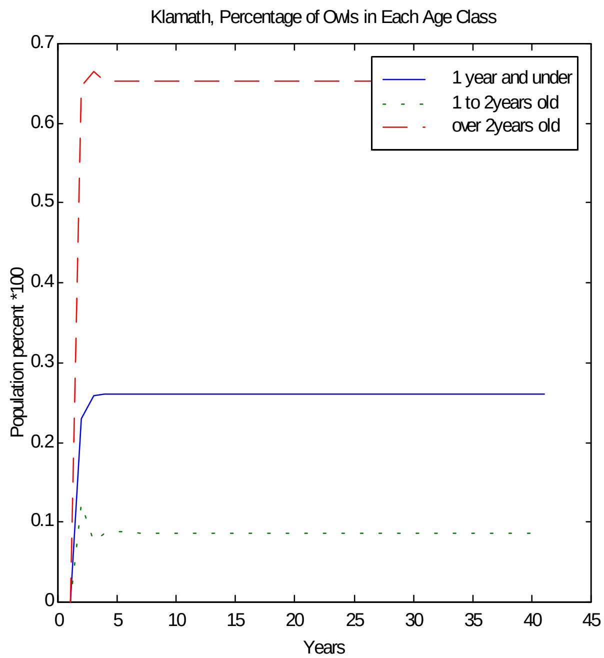
The Leslie-Lefkovitch matrix looked like this:

$$\begin{bmatrix} J_{t+1} \\ S_{t+1} \\ A_{t+1} \end{bmatrix} = \left[\begin{pmatrix} .09 & .20 & .33 \\ .325 & 0 & 0 \\ 0 & .8677 & .8677 \end{pmatrix} \right] \begin{bmatrix} 50 \\ 50 \\ 50 \end{bmatrix}$$
$$\lambda_1 = .9385$$

Applying this information to our matrix and using an initial age distribution vector of 50 Owls in each group we projected 25 years forward. This is what the graph looked like:



We then projected out 40 years and looked at the relative proportion of owls in each age class after a long period of time. The following graph shows the result.

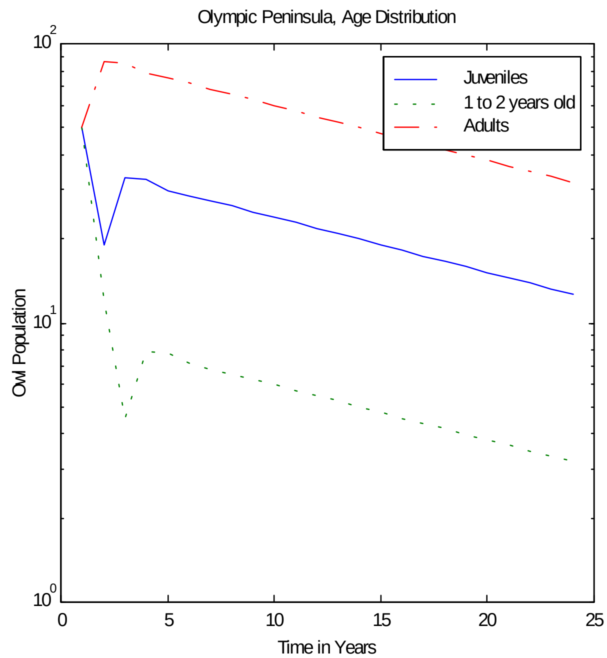


1.9.2 Olympic Peninsula, Washington

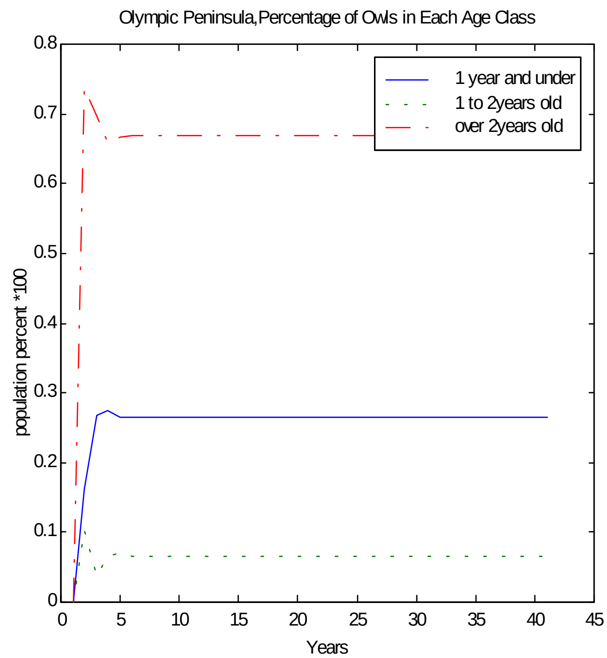
	Fecundity	Survivial
Juvenile	.0	.24
Subadults	.0	.86
Adults	.38	.87

$$\begin{bmatrix} J_{t+1} \\ S_{t+1} \\ A_{t+1} \end{bmatrix} = \left[\begin{pmatrix} .00 & .00 & .38 \\ .24 & 0 & 0 \\ 0 & .86 & .87 \end{pmatrix}\right] \begin{bmatrix} 50 \\ 50 \\ 50 \end{bmatrix}$$
$$\lambda_1 = .947$$

Projecting the Owl population 25 years delivered a graph like this:



The age distribution looked like this:

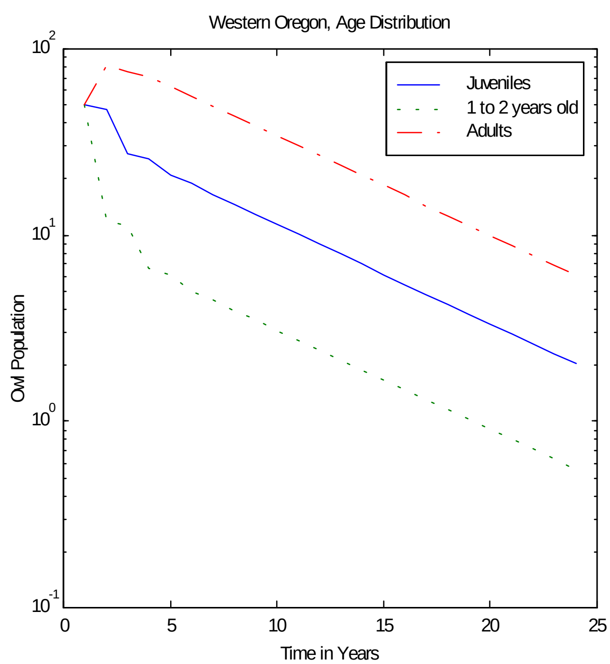


1.9.3 Coast Range, Western Oregon

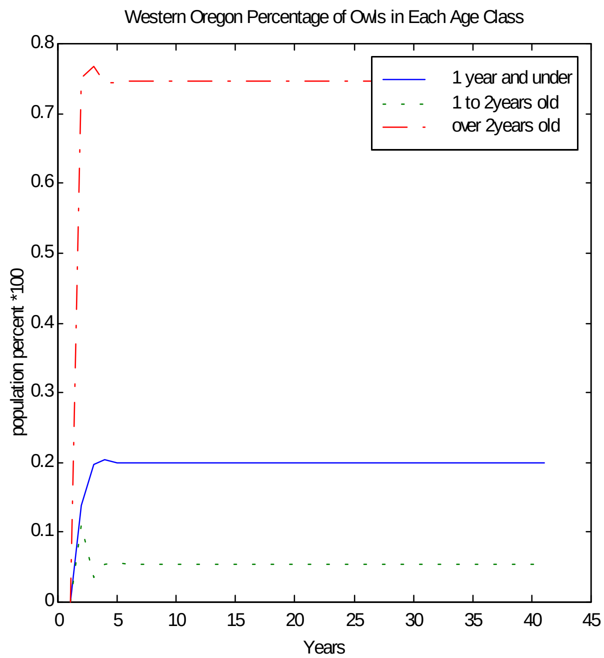
	Fecundity	Survivial
Juvenile	.0	.24
Subadults	.071	.82
Adults	.231	.81

$$\begin{bmatrix} J_{t+1} \\ S_{t+1} \\ A_{t+1} \end{bmatrix} = \left[\begin{pmatrix} .0 & .071 & .231 \\ .24 & 0 & 0 \\ 0 & .82 & .81 \end{pmatrix}\right] \begin{bmatrix} 50 \\ 50 \\ 50 \end{bmatrix}$$
$$\lambda_1 = .874$$

The Oregon data has a significant drop in both the adult fecundity rate and adult and subadult survival rates. Based on our projections the Owl population over 25 years would look like this:

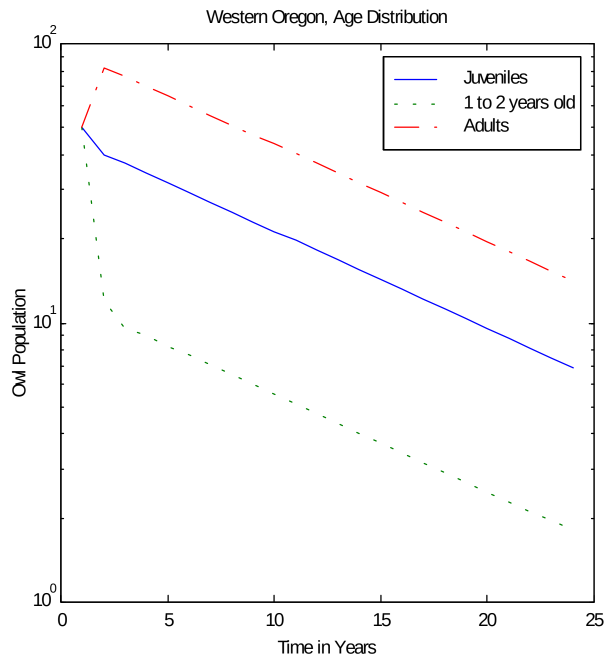


Age distribution:



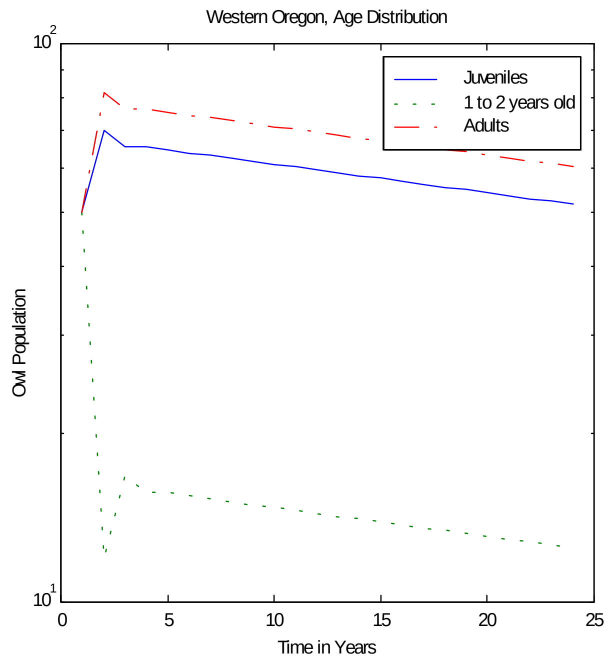
We then attempted to raise the Owl population in Oregon by raising the fecundity rate

	Fecundity	Survivial
Juvenile	.0	.24
Subadults	*.45*	.82
Adults	*.45*	.81



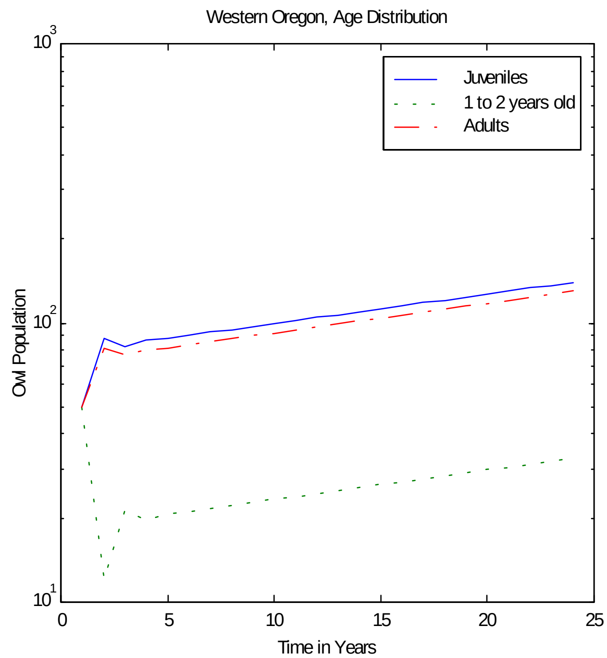
Rasing the fecundity rate again :

	Fecundity	Survivial
Juvenile	.0	.24
Subadults	*.70*	.82
Adults	*.70*	.81



Finally we raised the fecundity rate to show an increase in population.

	Fecundity	Survivial
Juvenile	.0	.24
Subadults	*..89*	.82
Adults	*.88*	.81



1.10 Conclusion

Using a Leslie-Lefkovitch matrix we modeled three areas with given information dealing with survival and fecundity rates. We observed that as these rates increased or decreased so did the populations over time. When we manipulated fecundity rates we were able to increase the population. . Another approach could be to examine the survival rate of the juveniles and our same model could be used. In our review of the three areas we found that a small change in fecundity rates effected the population outcome..

1.10.1 Literature Cited

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