Fractals

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Math 45 — College of the Redwoods

December 19, 1997

1 Introduction

Fractals can be used in many ways, to model coastlines, landscapes, plants, clouds, and diffusion of liquids into other liquids; the fractal generation algortithm can also be used in image compression.

A fractal is an image that is self-similar under magnification. Basically, if you zoom in on the image, you will see the whole image in the smaller part.

1.1 Generating Fractals Using Affine Transformations:

To understand how to generate a fractal image, it is necessary to be familiar with affine transformations. An affine transformation has the form:

$$y = A * x + T$$

where A is an nxn matrix of transformation, x is an $nx1$ vector, and T is an $nx1$ vector. For our purposes, this equation would look like this:

$$\begin{bmatrix} x_{n+1} \\ y_{n+1} \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} * \begin{bmatrix} x_n \\ y_n \end{bmatrix} + \begin{bmatrix} e \\ f \end{bmatrix} \quad (1)$$

This serves to transform an image and shift it.

Another necessary mathematical concept is matrices of rotation; they rotate a point about the origin. Given an angle θ, the matrix of rotation is:

$$\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$$

1.1.1 Now let's move on to the steps to find the affine transformations needed to generate a specific image:

1. Find an image you want to generate fractally (this could be any image, not necessarily one that is self-similar under magnification.)
2. Find a piece of the whole image that looks similar to the entire image.
3. Using guess and check or a computer program, find an affine transformation that will alter the whole image such that it resembles the part found in step 2.
4. Repeat steps 2 & 3 until the entire image can be covered by smaller representations of itself.

1.1.2 Now that you have a list of affine transformations, you can generate the original image fractically by using these steps:

1. Find any point X_0.
2. Substitute a random affine transformations from your list into equation #1 and solve for X_{n+1}.
3. Repeat step 2 ad infinitum (a couple hundred thousand times should suffice in real world applications).
4. Throw out the first 10 points, then plot the remaining points in the vector X.

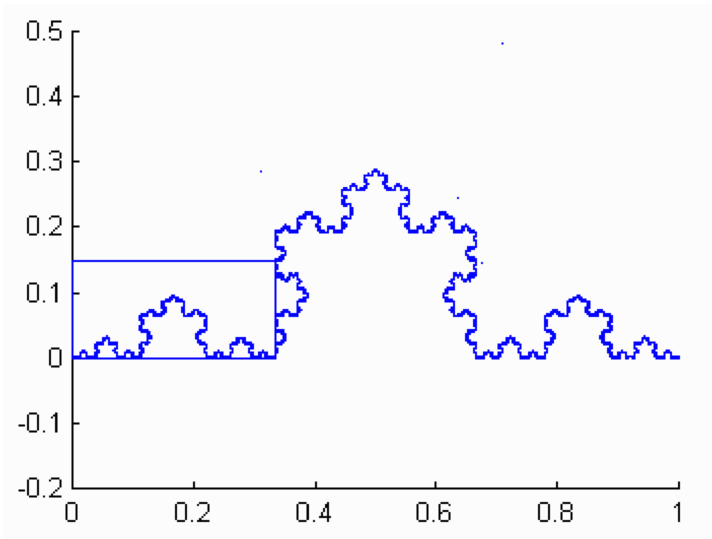

Figure 1:

1.2 GENERATING THE KOCH CURVE:

In 1904 Helge Von Kock, a Swedish mathematical, introduced the Koch cure or *snowflake curve*. He designed the cure because he wanted to find a curve that could not be differentiated. The Koch curve has become the foundation for the world of fractals.

Here is a picture illustrating the self-similarity of the koch curve, we chose a image generated fractally because it is an easy example to demonstrate, this process could be used on any image.

By zooming in on the area indicated by the red box in Figure 1 you will get Figure 2.

1.2.1 How we found the affine transformation to generate the Koch cure.

1. We stared with Figure 3, which is the basic skeleton of the Koch cure.

2. The first transformation will yield the blue part of Figure 3. Given By:

$$\begin{bmatrix} 1/3\cos 0 & -1/3\sin 0 \\ 1/3\sin 0 & 1/3\cos 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

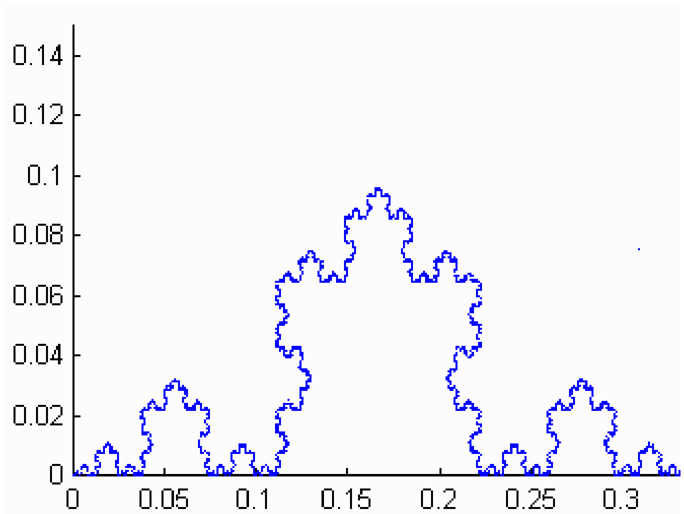

Figure 2:

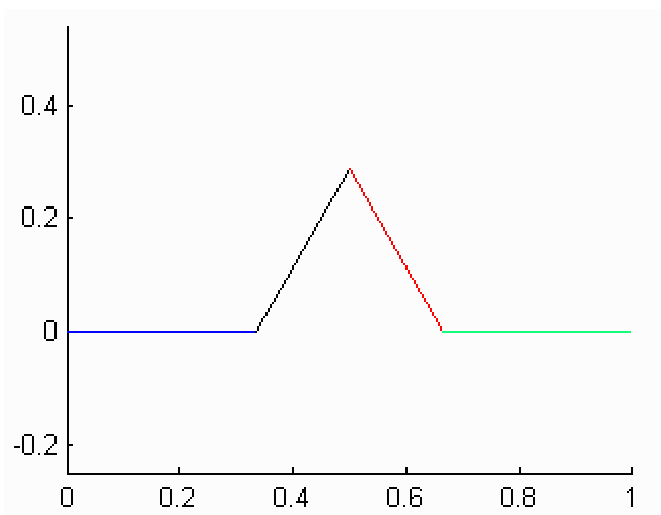

Figure 3:

which scales figure 3 by one-third, rotations it by zero degrees and shifts it by zero in the x & y direction.

3. The second transformation will yield the dark gray part of figure 3. Given By:

$$\begin{bmatrix} 1/3\cos 60 & -1/3\sin 60 \\ 1/3\sin 60 & 1/3\cos 60 \end{bmatrix} + \begin{bmatrix} 1/3 \\ 0 \end{bmatrix}$$

which scales figure 3 by one-third, rotations it by sixty degrees and shifts it by one-third in the x & zero in the
y direction.

4. The third transformation will yield the red part of figure 3. Given By:

$$\begin{bmatrix} 1/3\cos(-60) & -1/3\sin(-60) \\ 1/3\sin(-60) & 1/3\cos(-60) \end{bmatrix} + \begin{bmatrix} 1/2 \\ \frac{\sqrt{3}}{6} \end{bmatrix}$$

which scales figure 3 by one-third, rotations it by sixty degrees and shifts it by one-third in the x & zero in the
y direction.

5. The first transformation will yield the light green part of figure 3. Given By:

$$\begin{bmatrix} 1/3\cos 0 & -1/3\sin 0 \\ 1/3\sin 0 & 1/3\cos 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

which scales figure 3 by one-third, rotations it by zero degrees and shifts it by zero in the x & y direction.

1.3 References

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