

**Instructions.** (70 points) Place your solutions on your own paper.

(5<sup>pts</sup><sub>ea.</sub>) 1. Solve each of the following equations and/or inequalities for  $x$ .

(a)  $2x^{2/5} - 4x^{1/5} = 6$

(b)  $\left| x - \frac{x+1}{2} \right| < 3$

(c)  $\sqrt{3x-5} - \sqrt{x+7} = 2$

(d)  $\frac{5}{x+4} = 4 + \frac{3}{x-2}$

(5<sup>pts</sup>) 2. Consider the quadratic function  $f(x) = x^2 - 8x - 9$ . Perform each of the following tasks on graph paper.

(a) Use the method of completing the square to put the quadratic equation in vertex form. Plot and label the vertex with its coordinates. Draw the axis of symmetry and label it with its equation.

(b) Use a strictly algebraic approach (no calculators) to find the  $x$ - and  $y$ -intercepts of the quadratic function  $f$ . Plot and label each intercept with its coordinates. You must show your work to receive credit for this part.

(c) Sketch the graph of  $f$  using all of the information in parts (a) and (b). Use interval notation to describe the domain and range of the function  $f$ .

(5<sup>pts</sup>) 3. Consider the polynomial

$$p(x) = 2x^4 + 5x^3 - 17x^2 - 14x + 24.$$

Perform each of the following tasks.

(a) List all possible rational roots.

(b) Use synthetic division to find at least two roots from the list in part (a).

(c) Use the two roots found in part (b) to express  $p(x)$  as a product of two linear factors and one quadratic factor.

(d) Find the roots of the quadratic factor.

(e) State all of the roots of  $p(x)$ , then express  $p(x)$  as a product of linear factors.

(f) Sketch the graph of  $p(x)$ , then shade the solution of  $p(x) \geq 0$  on the  $x$ -axis. Describe the solution using interval notation.

4. Consider the polynomial function

$$p(x) = x^4 - 6x^3 - x^2 - 6x - 2.$$

Given that  $x = i$  is a root of  $p(x)$ , perform each of the following tasks.

(a) Find the remaining roots and express  $p(x)$  as a product of linear factors.

(b) Sketch the graph of  $p(x)$  and shade the solution of  $p(x) < 0$  on the  $x$ -axis. Use interval notation to describe the solution. Exact answers only. No decimals!

- (5<sup>pts</sup>) 5. Consider the function

$$f(x) = \frac{x^2 - 9}{x + 1}.$$

Perform each of the following tasks.

- Plot the  $x$ -intercepts and label them with their coordinates.
  - Plot the asymptotes and label them with their equations. Show the work supporting the finding of the asymptotes.
  - Sketch the graph of  $f$ .
  - Shade the solution of  $f(x) \leq 0$  on the  $x$ -axis and describe this solution using interval notation.
- (5<sup>pts</sup>) 6. For the following system, set up the augmented matrix, then use Gaussian Elimination and Back Substitution to find the solution.

$$\begin{aligned}x + 2y - 3z &= 4 \\2x + 5y - z &= -2 \\-x + 3y - z &= 0\end{aligned}$$

You will be marked down if you do not use the following:

- At the right of each row in the augmented matrix, use the appropriate symbolic notation to show the elementary row operation used to obtain the same row in the next matrix.
- When using Back Substitution, label each equation and reference each in the explanatory directions as you perform the Back substitution.
- Report your final answer in the form  $(x, y, z)$ .

7. Consider the system

$$\begin{aligned}2x + 3y - 6z &= 12 \\x + 4y + z &= 8\end{aligned}$$

Perform each of the following tasks.

- Write the augmented matrix for the system on your examination paper.
- Enter the augmented matrix into your calculator and use the appropriate command to find the reduced row echelon form of the matrix. Write this reduced form on your examination paper.
- State the solution of the system.

- (5pts) 8. Consider the function

$$f(x) = (x - 2)^2, \quad x \leq 2.$$

Note that the function is restricted to the domain  $(-\infty, 2]$ . Perform each of the following tasks.

- On graph paper, with each box representing 1 unit, sketch an accurate graph of the function  $f$ , plotting several points for accuracy. Then, on the same coordinate system, sketch the graph of  $f^{-1}$ , by reflecting the graph of  $f$  through the line  $y = x$ . Include the line  $y = x$  in your sketch.
- Use the technique of switching  $x$  and  $y$  to determine a formula for  $f^{-1}(x)$ . Use the graph from part (a) to help determine any choices you should make.
- Show that you have the true inverse by computing both  $f(f^{-1}(x))$  and  $f^{-1}(f(x))$ . Do not skip any steps in your work and justify any assumptions you make as you attempt to show that both of these compositions equal  $x$ .

- (5pts) 9. Given the function

$$f(x) = e^{-x} + 3,$$

perform each of the following tasks. When constructing your coordinate system, it is **required** that each box represent one unit.

- Use a purely algebraic approach to determine  $f^{-1}(x)$ .
- Using pencil, sketch the graph of  $f$ . Label the key point with its coordinates. Draw the asymptote as a dashed line and label it with its equation.
- Using a different colored pen or pencil, sketch the graph of  $f^{-1}$ . Label the key point with its coordinates. Draw the asymptote as a dashed line and label it with its equation. Points will be deducted if you do not use a different colored pen or pencil.
- Sketch the line  $y = x$  with a regular pencil. Check for appropriate symmetry before submitting your result.

10. Solve each of the following equations for  $x$ .

(a)  $\ln x - \ln(x + 3) = 2$

(b)  $5^{x-2} = 3^{3x+4}$

- (5pts) 11. Use your calculator to estimate  $\log_5 7$ , correct to the nearest ten-thousandth. Show any necessary preliminary work before providing the calculator result.

- (5pts) 12. Given that \$5,000 is invested at 5% per year compounded quarterly, find how much time it takes for the investment to double.

(5<sup>pts</sup>) **13.** A potato comes out of an oven with temperature  $450^\circ\text{F}$  and placed on a plate on the kitchen table where the room temperature is  $60^\circ\text{F}$ . In 3 minutes, the potato has cooled to  $250^\circ\text{F}$ . How long will it take the potato to cool to  $100^\circ\text{F}$ ?

**14.** Use Cramer's Rule to solve the system

$$\begin{aligned}x + 2y &= 6 \\ 3x - 4y &= 12.\end{aligned}$$

**15.** Solve the following equation for  $\lambda$ .

$$\begin{vmatrix} 0 & \lambda - 2\pi & 0 \\ \lambda - 2\pi & 0 & 0 \\ 1 & 0 & \lambda - 3 \end{vmatrix} = 0$$

**16.** Sum each of the following series.

(a)  $1 + 4 + 7 + \cdots + 1498$

(b)  $2 + 6 + 18 + \cdots + 118,098$

(c)  $0.8 + 0.03 + 0.003 + 0.0003 + \cdots$

(d)  $\sum_{k=1}^{\infty} 2(1/3)^{k-1}$

(5<sup>pts</sup>) **17.** Use the binomial theorem to expand  $(x - 2y)^6$ ,