Section 1.2

Quadratic Equations

Solving Quadratic Equations

$$ax^2 + bx + c = 0$$

In previous courses we solved quadratic equations by factoring and using the zero product property.

$$x^2 + 5x + 6 = 0$$

$$(x+2)(x+3) = 0$$

so $x = -2$ or $x = -3$

For most problems we will not be able to factor.

$$x^2 + 2x - 5 = 0$$

$x^2 + 2x - 5$ is prime.

Is there some other way to solve this equation?

YES!!

Let's start with some simple problems.

$$x^2 - 64 = 0$$
$$(x+8)(x-8) = 0$$
so $x = -8$ or $x = 8$

We can solve this a different way

If $x^2 - 64 = 0$ then $x^2 = 64$

then $x = \pm\sqrt{64} = \pm 8$

The Principle of Square Roots

If $x^2 = k$, then $x = \pm\sqrt{k}$ if $k \geq 0$

Find all solutions

$x^2 = 49$ $\quad$ $x^2 = -50$

$x^2 = 144$ $\quad$ $x^2 = -5$

The Principle of Square Roots

If $(x+p)^2 = k$, then $x + p = \pm\sqrt{k}$ if $k \geq 0$

so $x = \pm\sqrt{k} - p$ if $k \geq 0$

Solve for x

$$(x+3)^2 = 16$$
$$(x+3) = \pm 4$$
so $x = 4 - 3 = 1$ or $x = -3 - 3 = -7$

Find all solutions

$(x-2)^2 = 64$ $\qquad$ $(x+4)^2 = -36$

We can solve quadratic equations in the form

$$(x+3)^2 = 16$$
$$(x+3) = \pm\sqrt{16}$$
$$x = \pm 4 - 3$$
$$x = 1 \quad \text{or} \quad x = -7$$

Our goal is to put any quadratic equation in the form $(x+k)^2 = p$ before solving.

Solve the following

$$x^2 + 6x + 9 = 64$$
$$(x+3)^2 = 64$$
$$(x+3) = \pm\sqrt{64}$$
$$x = \pm 8 - 3$$
$$x = 5 \quad \text{or} \quad x = -11$$

If we can make the left hand side a perfect square trinomial, we can solve!!!

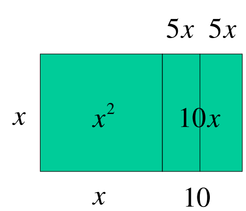
Recall $(x+3)^2 = x^2 + 6x + 9$ is a perfect square trinomial.

Notice that the 9 is the square of half of the coefficient of the x term.

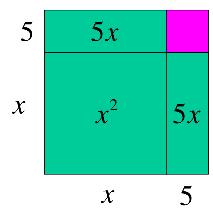
Given $x^2 + 10x$, how could we make this a perfect square trinomial?

We use a process called completing the square.

Comple the square on $x^2 + 10x$



Comple the square on $x^2 + 10x$



What do we need to complete the square?

Given $x^2 + 10x$, we need to add the square of half of the coefficient of the x term.

$$x^2 + 10x + 5^2 = x^2 + 10x + 25$$
$$= (x+5)^2$$

This process is called completing the square.

Given the expression $x^2 + bx$, complete the square by adding the square of half of the coefficient of the x term.

$$x^2 + bx + (b/2)^2 = \left(x + \frac{b}{2}\right)^2$$

The result is a perfect square trinomial.

$$\left(x + \frac{b}{2}\right)^2 = x^2 + 2\left(\frac{b}{2}x\right) + \left(\frac{b}{2}\right)^2$$

Solve the following by completing the square

$$x^2 + 6x - 7 = 0$$

$$x^2 + 6x = 7$$
$$+3^2 \quad +3^2$$

$$x^2 + 6x + 9 = 16$$

$$(x+3)^2 = 16$$

$$(x+3) = \pm\sqrt{16}$$

$$x = \pm 4 - 3$$

$$x = 1 \quad \text{or} \quad x = -7$$

Solve by completing the square

$$2x^2 + 8x - 64 = 0$$

$$x^2 + 4x - 32 = 0$$

$$x^2 + 4x = 32$$
$$+4 \quad +4$$

$$x^2 + 4x + 4 = 36$$

$$(x+2)^2 = 36$$

$$x + 2 = \pm\sqrt{36}$$

$$x = \pm 6 - 2$$

So $x = 4$ or $x = -8$

The Quadratic Formula

Solve the equation $ax^2 + bx + c = 0$ by completing the square.

$$ax^2+bx+c=0$$
$$ax^2+bx=-c$$
$$\frac{ax^2}{a}+\frac{bx}{a}=-\frac{c}{a}$$
$$x^2+\frac{b}{a}x=-\frac{c}{a}$$

To solve, we complete the square by adding

$\left(\frac{b}{2a}\right)^2$ to each side

$$x^2+\frac{b}{a}x+\frac{b^2}{4a^2}=-\frac{c}{a}+\frac{b^2}{4a^2}$$
$$x^2+\frac{b}{a}x+\frac{b^2}{4a^2}=-\frac{4ac}{4a^2}+\frac{b^2}{4a^2}$$
$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2}$$

$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2}$$
$$\left(x+\frac{b}{2a}\right)=\pm\sqrt{\frac{b^2-4ac}{4a^2}}$$
$$x+\frac{b}{2a}=\pm\frac{\sqrt{b^2-4ac}}{2a}$$
$$x=-\frac{b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$$

The Quadratic Formula

If $ax^2 + bx + c = 0$

then $x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Use the quadratic formula to solve

$x^2 - 6x + 9 = 0$

$y^2 + 4y = 21$

$x^2 = 8x - 16$

Graphical Solutions

Given a quadratic equation $ax^2 + bx + c = 0$

Graph $y = ax^2 + bx + c$

The solutions occur at the x-values where the graph of y intersects the x-axis.

There are three possibilities:

1) two real solutions, 2) one real solution, and 3) no real solutions

Analytical Solutions

Given a quadratic equation $ax^2 + bx + c = 0$

Use the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

If $b^2 - 4ac > 0$ there are two real solutions

If $b^2 - 4ac = 0$ there is one real solution

If $b^2 - 4ac < 0$ there are no real solutions.

Given the quadratic equation $ax^2 + bx + c = 0$

The expression $b^2 - 4ac$

is called the discriminant of the equation.

You can use the discriminant to quickly determine the number and types of solutions.

Use the discriminant to determine the types of solutions to the equation $3x^2 - 5x + 1 = 0$

Applications

The area of a rectangular window is 143 sq ft. If the length is 2 feet more than the width, find the dimensions of the window.

Let x be the width of the window, then $x + 2$ is the length.

Since the area is 143 sq ft, we get th equation

$x(x+2) = 143$

$$x^2 + 2x = 143$$

$$x^2 + 2x - 143 = 0$$

Using the quadratic formula

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, the positive solution is $x = 11$.

So the width is 11 ft, and the length is 13 ft.

Make sure to check the solutions!