

Section 3.5
Transformations of Graphs

Given the graph of $f(x)$ and $c > 0$

- $y=f(x)+c$ is the graph of $f(x)$ shifted c units upward.
- $y=f(x)-c$ is the graph of $f(x)$ shifted c units downward.
- $y=f(x-c)$ is the graph of $f(x)$ shifted c units to the right.
- $y=f(x+c)$ is the graph of $f(x)$ shifted c units to the left.

Examples

Graph $y = x^2 + 2$

Begin with the graph of $y = x^2$

Translate the graph 2 units up.

Describe the following graphs:

$$y = x^2 - 2$$

$$y = x^2 - \pi$$

$$y = x^2 + 5$$

Examples

Graph $y = \sqrt{x-2}$

Begin with the graph of $y = \sqrt{x}$

Translate the graph 2 units right.

Describe the following graphs:

$y = \sqrt{x+2}$

$y = \sqrt{x-\pi}$

$y = \sqrt{x+5}$

Graph $y = \frac{1}{x-4} + 2$

Begin with the graph of $y = \frac{1}{x}$

Move the graph 4 units to the right to get

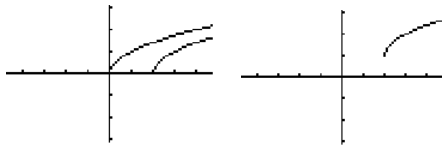
the graph of $y = \frac{1}{x-4}$

Move the graph 2 units up to get the

graph of $y = \frac{1}{x-4} + 2$

Examples

Describe the graph of $y = \sqrt{x-2} + 1$



Translate graph

of $y = \sqrt{x}$

right 2 units

Translate graph

up 1 unit

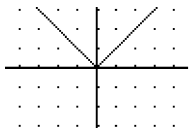
Given the graph of $f(x)$

Describe the graph of the following

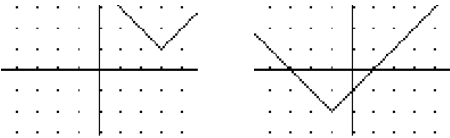
$f(x-2)+3$ $f(x-\pi)-\pi$

$f(x+2)-3$ $f(x+\pi)+\pi$

The graph of $f(x) = |x|$ is shown below.



Find an equation for each graph:



If (x, y) lies on the graph of $y = f(x)$, then (x, cy) lies on the graph of $y = c \cdot f(x)$

The graph of $y = c \cdot f(x)$ is obtained by vertically stretching the graph of $y = f(x)$ by a factor of c .

If $0 < c < 1$

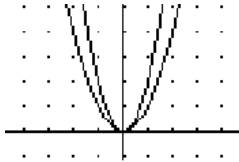
The graph of $y = c \cdot f(x)$ is obtained by vertically shrinking the graph of $y = f(x)$ by a factor of c .

Examples

Graph $y = 2x^2$

Begin with the graph of $y = x^2$

Vertically stretch the graph by a factor of 2.

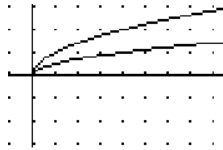


Examples

Graph $y = (1/2)\sqrt{x}$

Begin with the graph of $y = \sqrt{x}$

Vertically shrink the graph by a factor of 1/2.



Examples

Describe the graph of $y = 3|x|$

Describe the graph of $y = (1/5)x^3$

If (x, y) lies on the graph of $y = f(x)$, then

$\left(\frac{x}{c}, y\right)$ lies on the graph of $y = f(cx)$

If $c > 1$, then the graph of $y = f(c \cdot x)$ is obtained by horizontally shrinking the graph of $y = f(x)$.

If $0 < c < 1$

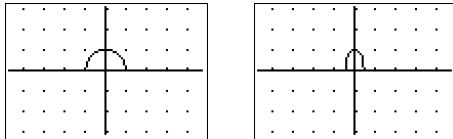
The graph of $y = f(c \cdot x)$ is obtained by horizontally stretching the graph of $y = f(x)$.

Examples

Graph $f(x) = \sqrt{1-x^2}$ and $f(2x) = \sqrt{1-(2x)^2}$

Begin with the graph of $y = \sqrt{1-x^2}$

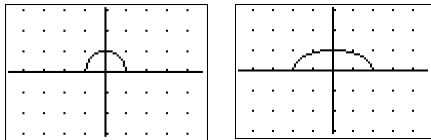
Horizontally shrink the graph by a factor of 2.



Graph $f(x) = \sqrt{1-x^2}$ and $f\left(\frac{1}{2}x\right) = \sqrt{1-\left(\frac{1}{2}x\right)^2}$

Begin with the graph of $y = \sqrt{1-x^2}$

Horizontally stretch the graph by a factor of 2.



Given the graph of $f(x)$.

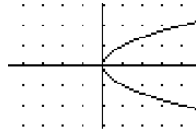
- $y=-f(x)$ is the graph of $f(x)$ reflected about the x -axis.
- $y=f(-x)$ is the graph of $f(x)$ reflected about the y -axis.

Examples

Graph $y = -\sqrt{x}$

Begin with the graph of $y = \sqrt{x}$

Reflect the graph about the x -axis to get
the graph of $y = -\sqrt{x}$

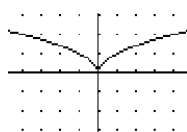


Examples

Graph $y = \sqrt{-x}$

Begin with the graph of $y = \sqrt{x}$

Reflect the graph about the y -axis to get
the graph of $y = \sqrt{-x}$



Examples

Graph $y = -\sqrt{-x}$

Begin with the graph of $y = \sqrt{x}$

Reflect the graph about the x -axis to get the graph of $y = -\sqrt{x}$

Reflect the graph about the y -axis to get the graph of $y = -\sqrt{-x}$

Examples

Graph $y = (x+3)^2 - 2$

Begin with the graph of $y = x^2$

Move the graph 3 units to the left to get the graph of $y = (x+3)^2$

Move the graph 2 units down to get the graph of $y = (x+3)^2 - 2$

Graph $y = \frac{1}{x-4} + 2$

Begin with the graph of $y = \frac{1}{x}$

Move the graph 4 units to the right to get the graph of $y = \frac{1}{x-4}$

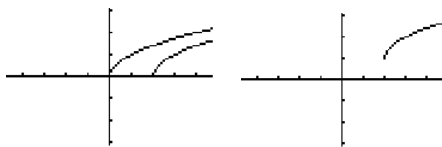
Move the graph 2 units up to get the graph of $y = \frac{1}{x-4} + 2$

Combining Transformations

- The following operations result in the same graph.
- Two Translations
- Two Reflections
- Translation in x direction and reflection in x -axis
- Translation in y -direction and reflection in y -axis

Examples

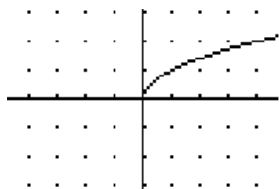
Describe the graph of $y = \sqrt{x-2} + 1$



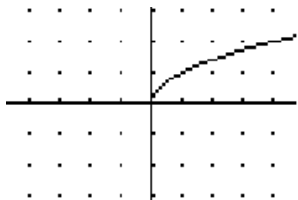
Translate graph
of $y = \sqrt{x}$
right 2 units

Translate graph
up 1 unit

Graph $y = -\sqrt{x-2}$ two ways



Graph $y = \sqrt{-x} + 1$ two ways



The following operations cannot be reversed

- Translation in y direction and reflection in x -axis
- Translation in x -direction and reflection in y -axis

Exceptions

Reflect about the x - axis first

Reflect about the y - axis last

x reflection before y translation

x translation before y reflection

Always do y translations last!

Case 1: A reflection about the x – axis
and a translation in the y direction.

Reflect about the x axis first, then
translate up or down

$$y = -|x| + 3$$

Reflect $|x|$ about the x axis first,
then translate up 3 units

$$y = -\sqrt{x} - 4$$

Reflect \sqrt{x} about the x axis first,
then translate down 4 units

Case 2: A translation in the x direction and
a reflection about the y – axis.

Translate left or right first, then
reflect about the y axis

$$y = |-x + 3|$$

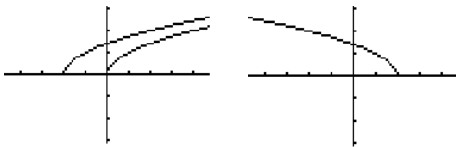
Translate $|x|$ left 3 units then
reflect about the y axis

$$y = \sqrt{-x-4}$$

Translate \sqrt{x} right 4 units then reflect about the y axis

Examples

Describe the graph of $y = \sqrt{-x+2}$

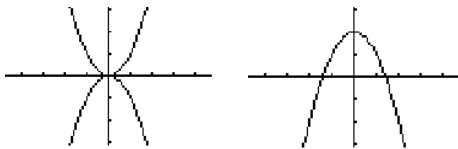


Translate graph of $y = \sqrt{x}$ left 2 units

Reflect graph across y-axis

Examples

Describe the graph of $y = -x^2 + 2$



Reflect graph of $y = x^2$ across x-axis

Translate graph up 2 units

$$y = -\sqrt{-x+4} + 1$$

Reflect \sqrt{x} about x axis

translate left 4 units

reflect about the y axis

translate up 1

$$y = -(-x-2)^2 - 3$$

Reflect x^2 about x axis

translate right 2 units

reflect about the y axis

translate down 3
