

Exponential Functions
Section 5.7

Compound Interest

Simple Interest Formula

$$I = P \cdot r \cdot t$$

I = Interest earned

P = Principal amount invested

r = annual interest rate as a decimal

t = length of time in years

Suppose you invest \$1,000 for 1 year at an 8% annual interest, where the interest is compounded quarterly. How much money will you have at the end of the year?

Compound Interest Formula

$$A = P \left(1 + \frac{r}{n} \right)^{nt}$$

A = Amount of money returned

P = Principal amount invested

r = annual interest rate as a decimal

t = length of time in years

n = number of compoundings per year

Suppose you invest \$1,000 for 1 year at an 8% annual interest, but the interest is compounded quarterly. How much money will you have at the end of the year?

Use the compound interest formula.

Simplified Compound Interest Formula

$$A = P(1 + r)^n$$

A = Amount of money returned

P = Principal amount invested

r = periodic interest rate as a decimal

n = number of periodic interest payments

Suppose you invest \$100 for 1 year at an 8% annual interest, but the interest is compounded monthly. How much money will you have at the end of the year?

Use the compound interest formula.

Suppose you invest \$100 for 1 year at an 8% annual interest, but the interest is compounded daily. How much money will you have at the end of the year?

Use the compound interest formula.

Suppose you invest \$100 for 1 year at an 8% annual interest, but the interest is compounded hourly. How much money will you have at the end of the year?

Use the compound interest formula.

Suppose you invest \$100 for 1 year at an 8% annual interest, but the interest is compounded every second. How much money will you have at the end of the year?

Use the compound interest formula.

The term $\left(1 + \frac{1}{n}\right)^n$ is very important.

Evaluate this term for several large values of n .

Definition

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e$$

$$e \approx 2.71828$$

Use your calculator to graph $y = \left(1 + \frac{1}{x}\right)^x$

What happens to y as x get very large?

Compound Interest Formula

$$A = P \left(1 + \frac{r}{n}\right)^{nt}$$

If we let $k = \frac{n}{r}$, then $n = rk$ and $\frac{1}{k} = \frac{r}{n}$ and we get

$$A = P \left(1 + \frac{r}{n}\right)^{nt} = P \left(1 + \frac{1}{k}\right)^{rkt} = P \left[\left(1 + \frac{1}{k}\right)^k\right]^{rt}$$

If $n \rightarrow \infty$, then $k \rightarrow \infty$

$$\text{so } \left(1 + \frac{1}{k}\right)^k = e \quad \text{so } P \left[\left(1 + \frac{1}{k}\right)^k\right]^{rt} = Pe^{rt}$$

Continuous Compound Interest Formula

$$A = Pe^{rt}$$

A is the amount of money returned

P is the principal invested

r is the annual interest rate as a decimal

t is the number of years for the investment

If \$10,000 is deposited in an account that pays 6% compounded continuously for 4 years, how much money will be in the account after 4 years?

If \$10,000 is deposited in an account that pays 6% per year, how much money will be in the account after 4 years if the interest is compounded monthly?

Solve the compound interest formula
for P

$$A = P \left(1 + \frac{r}{n} \right)^{nt}$$

$$\frac{A}{\left(1 + \frac{r}{n} \right)^{nt}} = P \frac{\left(1 + \frac{r}{n} \right)^{nt}}{\left(1 + \frac{r}{n} \right)^{nt}} \quad \text{so } P = A \cdot \left(1 + \frac{r}{n} \right)^{-nt}$$

$P = A \left(1 + \frac{r}{n} \right)^{-nt}$ is called the Present Value Formula

P is the present value of A dollars to be received after t years assuming a per annum interest rate of r compounded n times per year.

How much money would you need to invest now if you wanted to have \$10,000 in fifteen years assuming an interest rate of 8% compounded monthly?

$$\text{Since } P = A \left(1 + \frac{r}{n}\right)^{-nt}$$

$$A = 10000, \quad r = .08, \quad n = 12, \quad \text{and } t = 15$$

$$P = 10000 \left(1 + \frac{.08}{12}\right)^{-12 \cdot 15}$$

$$P \approx \$3023.96$$

Check this using the compound interest formula

Suppose you invest \$1000 and you want to have \$2000 in five years. What annual interest rate would you need if the interest is compounded once per year?

We use the formula $A = P(1+r)^n$ and solve for r where $A = 2000$, $P = 1000$, $n = 5$

so $2000 = 1000(1+r)^5$ now solve for r

$$2 = (1+r)^5$$

$$\sqrt[5]{2} = (1+r)$$

$$r = \sqrt[5]{2} - 1 \approx 0.148698 \approx 14.87\%$$
