

Short Answer Problems (50 points) Directions must be followed in order to receive credit! You must work ALONE on this exam. Do not consult with anyone other than your instructor. Work each problem on your own graph paper. Make sure that your work is neat and legible. Use a straightedge when drawing graphs. Be sure to show all necessary work to receive credit. Place this signed exam paper on top of your worked pages and staple them together. This exam is due on **Monday July 20. Late submissions will NOT be accepted.** By signing below you agree the work in this exam is solely your own. You agree that you have not discussed this exam nor received help from anyone with the exception of your instructor.

Signature: _____

- (5pts) 1. Consider the quadratic function

$$f(x) = 2x^2 - 7x - 15$$

Perform each of the following tasks.

- (a) Put f in vertex form using the process of completing the square.
- (b) State the vertex and the axis of symmetry and plot these on a properly labeled coordinate system.
- (c) Find the zeros of the function by factoring and then plot them on the coordinate system.
- (d) Find and plot the y-intercept. Then use the axis of symmetry to plot another point.
- (e) Complete the graph of f .

- (3pts) 2. A park ranger with 300 meters of fencing wants to enclose a rectangular plot that borders on a river. If no fence is required along the river, what is the largest area that can be enclosed?

- (3pts) 3. Find all real solutions, if any, of the equation

$$12x^3 + 16x^2 - 36 = 27x$$

- (5pts) 4. Consider the function

$$f(x) = 2x^3 + 3x^2 - 8x - 12$$

Perform each of the following tasks.

- (a) Determine the end behavior of the function without the use of a calculator. Explain your answer.
- (b) Find the y-intercept and the zeros of the function without the use of a calculator. You must show your work.
- (c) Using your calculator find all maxima and/or minima of the function. Determine whether the extrema found are local or absolute.
- (d) Sketch a nice graph of f .

(5pts) 5. Consider the function

$$f(x) = -\frac{2}{x+6} + 3$$

Perform each of the following tasks.

- (a) Analytically find the equation of the vertical asymptote of f . Explain your answer.
- (b) Find the equation of the horizontal asymptote of f . Explain your answer.
- (c) State the domain of f in set-builder notation.
- (d) Sketch a nice graph of f .

(5pts) 6. Consider the function

$$f(x) = \frac{x^2 - 81}{x^2 - 4x - 45}$$

Perform each of the following tasks.

- (a) Analytically find the equation of the vertical asymptote and any holes that may exist for the function f . Explain your answer.
- (b) Find the equation of the horizontal asymptote of f . Explain your answer.
- (c) State the domain of f in set-builder notation.
- (d) State the range of f in set-builder notation.
- (e) Sketch a nice graph of f .

(2pts) 7. Suppose $f(x) = \frac{8 - 7x - x^2}{x^2 - 8x - 9}$ and $g(x) = \frac{x^2 - 6x - 7}{x^2 - 6x + 5}$.

Compute $f(x) \cdot g(x)$ and simplify your answer.

(2pts) 8. Suppose $f(x) = \frac{x^2 + 15x + 56}{x^2 - x - 20}$ and $g(x) = \frac{-7 - x}{x + 1}$.

Compute $f(x) \div g(x)$ and simplify your answer.

(4pts) 9. Suppose $f(x) = \frac{4x}{-x - x^2}$ and $g(x) = \frac{4}{x^2 + 3x + 2}$.

(a) Compute $f(x) + g(x)$ and simplify your answer.

(b) Compute $f(x) - g(x)$ and simplify your answer.

(5pts) 10. Algebraically solve the following equation:

$$\frac{x}{x - 7} - \frac{8}{5 - x} = \frac{x + 7}{x^2 - 12x + 35}$$

(5pts) 11. Izzy can swim 2 miles per hour. If she swims 4 miles up stream in a river in the same time it takes her to swim 10 miles downstream, what is the speed of the river's current?

(6pts) 12. Chai Chai is going to make herself a new doggie bed out of a rectangular piece of cardboard that measures 8 inches by 12 inches. She cuts four smaller squares from each corner of the cardboard square, tossing the material aside. She then bends up the sides of the remaining cardboard to form an open box with no top that she can use as a bed. Find the dimensions of the squares cut from each corner of the original piece of cardboard so that Chai Chai maximizes the volume of the resulting box. Do each of the following:

(a) Draw and label a picture of the problem.

(b) Set up an equation that determines the volume of Chai's bed.

(c) State the empirical domain for the volume function.

(d) Graph this function over the empirical domain on your calculator and carefully draw the graph on your paper.

(e) Find the maximum volume of Chai's bed.

(f) What are the dimensions of the small squares that Chai cut out of the cardboard?