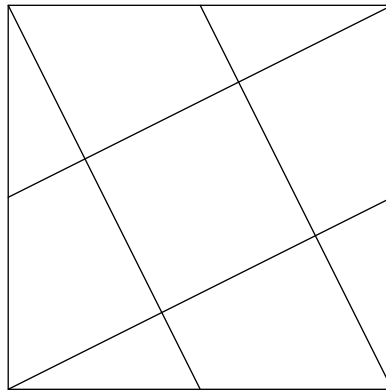


Pizza and Problem Solving

College of the Redwoods
Mathematics Department

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1. Lines from the vertices of a square to the midpoints of the sides are drawn, as shown below. Prove the surprising result that the area of the smaller square this produces is $1/5$ the given square.



2. Solve the system of equations

$$\begin{aligned}x + y + z &= a \\x^2 + y^2 + z^2 &= b^2 \\xy &= z^2,\end{aligned}$$

for x , y , and z , where a and b are constants.

3. The function f satisfies the functional equation

$$f(x) + f(y) = f(x + y) - xy - 1,$$

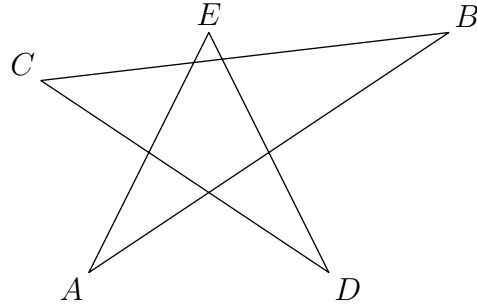
for every pair x, y of real numbers. If $f(1) = 1$, how many integers n (with $n \neq 1$) are there such that $f(n) = n$?

4. What is the probability that the quadratic equation

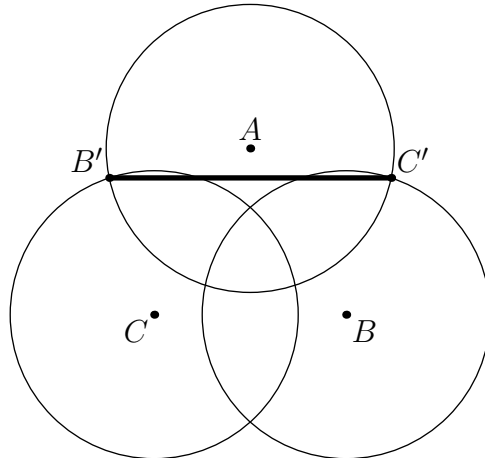
$$x^2 + bx + c = 0$$

has real roots?

5. Prove that the sum of the vertex angles (the angles at A , B , C , D , and E) of the five pointed star shown below is 180° .



6. Prove that there exists a power of three that ends with 001.
7. Circles with centers A , B and C each have radius r , where $1 < r < 2$. The distance between each pair of centers is 2. If B' is the point of intersection of circle A and circle C which is outside circle B , and if C' is the point of intersection of circle A and circle B which is outside circle C , find the length of $B'C'$.



8. For each positive number x , let

$$f(x) = \frac{\left(x + \frac{1}{x}\right)^6 - \left(x^6 + \frac{1}{x^6}\right) - 2}{\left(x + \frac{1}{x}\right)^3 + \left(x^3 + \frac{1}{x^3}\right)}.$$

What is the minimum value of $f(x)$?